WELCOME

Modified and Multiplicative Zagreb Indices on Graph Operators of Some Standard Graphs

Dr. L. Benedict Michael Raj
Department of Mathematics
St. Joseph's College(Autonomous), Trichy-2

13^{th} August, 2016

INTRODUCTION

INTRODUCTION

Zagreb indices are important topological indices in mathematical chemistry. The Zagreb indices have been introduced by Gutman and Trinajstic.

INTRODUCTION

Zagreb indices are important topological indices in mathematical chemistry. The Zagreb indices have been introduced by Gutman and Trinajstic.

For a graph $G = (V(G), E(G))$, the first and the second Zagreb indices were defined as $M_1(G) = \sum_{uv \in E(G)} d(u) + d(v)$ and $M_2(G) = \sum_{uv \in E(G)} d(u)d(v)$ respectively, where $d(u)$ denotes the degree of the vertex u in G.

In addition to the original Zagreb indices, several modified versions and new version of Zagreb indices thereof were also introduced and studied.

In addition to the original Zagreb indices, several modified versions and new version of Zagreb indices thereof were also introduced and studied.

Nikolic *et al.* introduced modified Zagreb indices. The first and the second modified Zagreb index were defined as
$^{m}M_1(G) = \sum_{v\in V(G)} \frac{1}{(d(v))^2}$ and $^{m}M_2(G) = \sum_{uv\in E(G)} \frac{1}{d(u)d(v)}$,
where $d(v)$ is the degree of the vertex v in G.

Todeschini *et al.* have suggested to consider multiplicative variants of additive graph invariants, which applied to the Zagreb indices would lead to the multiplicative Zagreb indices of a graph G, denoted by $\Pi_1(G)$ and $\Pi_2(G)$, under the name first and second multiplicative Zagreb index, respectively.

Todeschini *et al.* have suggested to consider multiplicative variants of additive graph invariants, which applied to the Zagreb indices would lead to the multiplicative Zagreb indices of a graph G, denoted by $\Pi_1(G)$ and $\Pi_2(G)$, under the name first and second multiplicative Zagreb index, respectively.

$\Pi_1(G) = \prod_{v \in V(G)} d(v)^2$ and $\Pi_2(G) = \prod_{uv \in E(G)} d(u)d(v)$, where $d(u)$ denotes the degree of the vertex u in G.

GRAPH OPERATORS

PRELIMINARIES

GRAPH OPERATORS

- Cvetkocic defined the *subdivision graph* $S(G)$ as the graph obtained from G by replacing each of its edge by a path of length 2, or equivalently by inserting an additional vertex into each edge of G.
- The operator $R(G)$ is the graph obtained from G by adding a new vertex corresponding to each edge G and by joining each new vertex to the end vertices of the edge corresponding to it.

STANDARD GRAPHS

TURNIP GRAPHS

A turnip $Tu_{n,g}$ is a graph obtained from a cycle on g vertices by attaching $n - g$ pendant edges to one of its vertices.

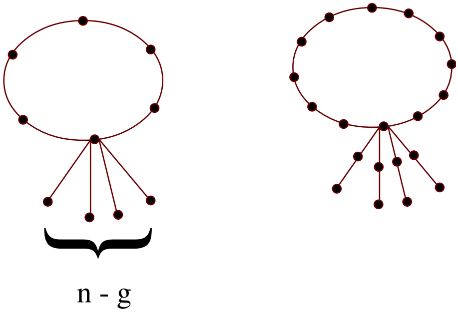

Figure: Turnip graph, $Tu_{10,6}$ and its subdivision $S(Tu_{10,6})$

KITE GRAPHS

A kite $Ki_{n,w}$ is a graph obtained from a clique on w vertices by attaching a path on $n-w$ vertices to one of its vertices.

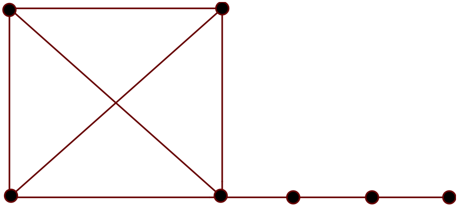

Figure: Kite graph, $Ki_{8,4}$

BAG GRAPHS

A bag $Bag_{p,q}$ is a graph obtained from a complete graph K_p by replacing an edge uv by a path P_q.

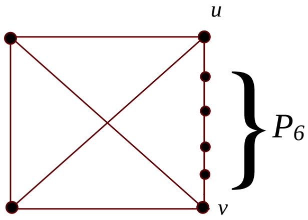

Figure: Bag graph, $B_{4,6}$

MAIN RESULTS

Modified and multiplicative Zagreb indices of $S(G)$ for some standard graphs

Lemma

The first modified and multiplicative Zagreb indices for the turnip graph are given by ${}^{m}M_1(Tu_{n,g}) = \frac{g-1}{4} + \frac{1}{(n-g+2)^2} + n - g$ *and*
$\Pi_1(Tu_{n,g}) = 4(g-1)(n-g+2)^2(n-g)$.

Proof.

The Turnip graph $Tu_{n,g}$ contains $g-1$ vertices of degree 2, one vertex of degree $n-g+2$ and $n-g$ pendant vertices. Hence, we get ${}^{m}M_1(Tu_{n,g}) = \frac{g-1}{4} + \frac{1}{(n-g+2)^2} + n - g$ and
$\Pi_1(Tu_{n,g}) = 4(g-1)(n-g+2)^2(n-g)$. ∎

Lemma

The second modified and multiplicative Zagreb indices for the turnip graph are given by ${}^{m}M_2(Tu_{n,g}) = \dfrac{ng - g^2 + 2n}{4(n-g+2)}$ *and* $\Pi_2(Tu_{n,g}) = 16(g-2)(n-g+2)(n-g)$.

Proof.

The Turnip has $g-2$ pairs of vertices of degree 2, two pairs of vertices of degree 2 and $n-g+2$ and $n-g$ pairs of vertices of degree 1 and $n-g+2$. Therefore, ${}^{m}M_2(Tu_{n,g}) = \dfrac{ng - g^2 + 2n}{4(n-g+2)}$ and $\Pi_2(Tu_{n,g}) = 16(g-2)(n-g+2)(n-g)$. ■

Theorem

The first modified and multiplicative Zagreb indices of subdivision for the turnip graph are given by

$$^{m}M_1(S(Tu_{n,g})) = ^{m}M_1(Tu_{n,g}) + n(n-g+2)^2 \text{ and}$$

$$\Pi_1(S(Tu_{n,g})) = \Pi_1(Tu_{n,g}) \left(\frac{n+g-1}{g-1}\right) \left(\frac{n-g+1}{n-g+2}\right)^2.$$

Theorem

The second modified and multiplicative Zagreb indices of subdivision for the turnip graph are given by

$$^{m}M_2(S(Tu_{n,g})) = {}^{m}M_2(Tu_{n,g}) + \frac{2n^2+g^2+2n-3ng}{4(n-g+2)}$$ *and*

$$\Pi_2(S(Tu_{n,g})) = 2\Pi_2(Tu_{n,g})\frac{(g-1)(n-g+2)}{g-2}.$$

Lemma

The first modified and multiplicative Zagreb indices for the kite graph are given by ${}^{m}M_1(ki_{n,w}) = \frac{n-w+1}{4} + \frac{10}{9}$ *and* $\Pi_1(ki_{n,w}) = 36(n-w+1)$, *for* $w = 3$. ${}^{m}M_1(ki_{n,w}) = \frac{n-w+3}{4} + \frac{w^2+w-1}{w^2(w-1)}$ *and* $\Pi_1(ki_{n,w}) = 4(n-w-1)w^2(w-1)^3$, *for* $w \geq 4$

Proof.

The kite graph reduces to path when $w = 2$. If $w = 3$, $ki_{n,w}$ has $n - w + 1$ vertices of degree 2, a vertex of degree 3 and a pendant vertex. Hence ${}^{m}M_1(ki_{n,w}) = \frac{n-w+1}{4} + \frac{10}{9}$ and $\Pi_1(ki_{n,w}) = 36(n-w+1)$. If $w \geq 4$, $ki_{n,w}$ has $n - w - 1$ vertices of degree 2, $w - 1$ vertices of degree $w - 1$, a vertex of degree w and a pendant vertex. Therefore, ${}^{m}M_1(ki_{n,w}) = \frac{n-w+3}{4} + \frac{w^2+w-1}{w^2(w-1)}$ and $\Pi_1(ki_{n,w}) = 4(n-w-1)w^2(w-1)^3$. ∎

Lemma

The second modified and multiplicative Zagreb indices for the kite graph are given by $^{m}M_2(ki_{n,w}) = \dfrac{n-w+3}{4}$ *and* $\Pi_2(ki_{n,w}) = 2^4 3^2 (n-w-1)$, *for* $w = 3$. $^{m}M_2(ki_{n,w}) = \dfrac{n-w}{4} + \dfrac{4w-5}{2w(w-1)}$ *and* $\Pi_2(ki_{n,w}) = 2^3 w^2 (n-w-2)(w-1)^5(w-2)$, *for* $w \geq 4$.

Proof.

If $w = 3$, $ki_{n,w}$ has $n-w-1$ pairs of vertices of degree 2, a pair of vertices of degree 1 and 2, 3 pairs of vertices of degree 2 and 3. Hence $^{m}M_2(ki_{n,w}) = \dfrac{n-w+3}{4}$ and $\Pi_2(ki_{n,w}) = 2^4 3^2 (n-w-1)$. If $w \geq 4$, $ki_{n,w}$ has $n-w-2$ vertices of degree 2, $\dfrac{(w-2)(w-1)}{2}$ pairs of vertices of degree $w-1$, $w-1$ pairs of vertices of degree w and $w-1$, one pair of vertices of degree 1 and 2 and one pair of vertices of degree w and 2. Hence $^{m}M_2(ki_{n,w}) = \dfrac{n-w}{4} + \dfrac{4w-5}{2w(w-1)}$ and

Theorem

The first modified and multiplicative Zagreb indices of subdivision for the kite graph are given by

$$^{m}M_1(S(ki_{n,w})) = ^{m}M_1(ki_{n,w}) + \frac{w^2+2n-3w+10}{8} \text{ and}$$

$$\Pi_1(S(ki_{n,w})) = \Pi_2(ki_{n,w})\frac{w^2+3w+4n+2}{(w-1)(n-w-1)}$$

Theorem

The second modified and multiplicative Zagreb indices of subdivision for the kite graph are given by

$$^{m}M_2(S(ki_{n,w})) = \frac{n-w+4}{2} \text{ and}$$

$$\Pi_2(S(ki_{n,w})) = \Pi_2(ki_{n,w})\frac{3(w-1)(n-w-1)}{8(4w+nw-w^2-1)}.$$

Lemma

The first modified and multiplicative Zagreb indices for the bag graph are given by ${}^{m}M_1(Bag_{p,q}) = \frac{q-2}{4} + \frac{p}{(p-1)^2}$ *and* $\Pi_1(Bag_{p,q}) = 4p(p-1)^2(q-2)$.

Proof.

The bag graph $Bag_{p,q}$ has $q-2$ vertices of degree 2 and p vertices of degree $p-1$. Hence ${}^{m}M_1(Bag_{p,q}) = \frac{q-2}{4} + \frac{p}{(p-1)^2}$ and $\Pi_1(Bag_{p,q}) = 4p(p-1)^2(q-2)$. ∎

Lemma

The second modified and multiplicative Zagreb indices for the bag graph are given by

$${}^{m}M_2(Bag_{p,q}) = \frac{p^2(q-1) + 2p(4-q) + q(p^2+1) - 11}{4(p-1)^2} \text{ and}$$

$$\Pi_2(Bag_{p,q}) = 4(p-1)^3(p^2-p-2)(q-3)^2$$

Proof.

The bag graph $Bag_{p,q}$ has $\frac{p(p-1)}{2} - 1$ pairs of vertices of degree $p-1$ and two pairs of vertices of degree 2 and $p-1$. Hence we have

$${}^{m}M_2(Bag_{p,q}) = \frac{p^2(q-1) + 2p(4-q) + q(p^2+1) - 11}{4(p-1)^2}$$

$$\Pi_2(Bag_{p,q}) = 4(p-1)^3(p^2-p-2)(q-3)^2$$

■

Theorem

The first modified and multiplicative Zagreb indices of subdivision for the bag graph are given by

$^{m}M_1(S(Bag_{p,q})) = ^{m}M_1(Bag_{p,q}) + \frac{p(p-1)+2(q-2)}{8}$ *and*

$\Pi_1(S(Bag_{p,q})) = \Pi_1(Bag_{p,q})\frac{p(p-1)+4(q-2)}{2(q-2)}$

Theorem

The second modified and multiplicative Zagreb indices of subdivision for the bag graph are given by

$^{m}M_2(S(Bag_{p,q})) = \frac{2p(p-1)+2(q-2)(p-1)}{4(p-1)}$ *and*

$\Pi_2(S(Bag_{p,q})) = \frac{p(q-2)}{4}$.

Modified and multiplicative Zagreb indices of $R(G)$ in terms of $S(G)$ for some standard graphs

Here, we obtain modified and multiplicative Zagreb indices of $R(G)$ in terms of $S(G)$ for turnip graph, kite graph and bag graph.

Theorem

The first modified and multiplicative Zagreb indices of $R(G)$ for the turnip graph are given by

$$^{m}M_1(R(Tu_{n,g})) = \frac{4 + 4(2n-g)(n-g+2)^2 + (g-1)(n-g+2)^2}{16(n-g+2)^2} \text{ and}$$

$$\Pi_1(R(Tu_{n,g})) = 64\Pi_1(Tu_{n,g})\frac{2n-g}{n-g}.$$

Theorem

The second modified and multiplicative Zagreb indices of $R(G)$ for the turnip graph are given by
$$^{m}M_2(R(Tu_{n,g})) = \frac{n(9n-7g+10)+3g(g-2)}{16(n-g+2)}$$ *and*
$$\Pi_2(R(Tu_{n,g})) = 8192\Pi_2(Tu_{n,g})(g-1)(n-g+1).$$

Theorem

The first modified and multiplicative Zagreb indices of $R(G)$ for the kite graph are given by $^{m}M_1(R(ki_{n,w})) =$
$$\frac{w^2-3w+2n+2}{4} + \frac{1}{4w^2} + \frac{n-w-1}{16} + \frac{w-1}{4(w-1)^2}$$ *and*
$$\Pi_1(R(ki_{n,w})) = \frac{(w^2-3w+2n+2)(n-w-1)}{2^{11}w^2(w-1)}.$$

Theorem

The second modified and multiplicative Zagreb indices of $R(G)$ for the kite graph are given by

$${}^mM_2(R(ki_{n,w})) = \frac{2nw(w-1)+3(w-1)+2w}{8w(w-1)} \text{ and}$$

$$\Pi_2(R(ki_{n,w})) = 2^6w^2 3(n-w-1)(w-1)^2.$$

Theorem

The first modified and multiplicative Zagreb indices of $R(G)$ for the bag graph are given by

$${}^mM_1(R(Bag_{p,q})) = \frac{2(p-1)^2(p^2-p+2q-4)+(q-2)(p-1)^2+4p}{16(p-1^2)} \text{ and}$$

$$\Pi_1(R(Bag_{p,q})) = 2^7[p^2(p-1)^3(q-2)+(2q-4)(q-2)(p-1)^2].$$

Theorem

The second modified and multiplicative Zagreb indices of $R(G)$ for the bag graph are given by

$$^{m}M_2(R(Bag_{p,q})) = \frac{2(q-2)(p-1)^2 + p(p-1) - 2 + 2p(p-1)^2 + 3(p-1)}{8(p-1^2)} \text{ and}$$

$$\Pi_2(R(Bag_{p,q})) = \Pi_2(S(Bag_{p,q}))2^{11}(p-1)^5(p^2-p-2).$$

REFERENCES

REFERENCES

- Cvetkocic D. M, Doob M, Sachs H, *Spectra of graphs - Theory and Application*, Academic Press, New York, 1980.
- Dhanalakshmi K, Amalorpava Jerline J, Benedict Michaelraj L, Modified and Multiplicative Zagreb Indices on Graph Operators, Journal of Computer and Mathematical Sciences-7(4), (2016) 225-232.
- Ghorbani M, Hosseinzadeh M. A, *A note on Zagreb indices of nanostar dendrimers*, Optoelectron.Adv. Mater., 4(2010), 1887-1880.
- Ghorbani M, Hosseinzadeh M. A, *A new version of Zagreb indices*, Filomat, 26(2012),93-100.
- Li S, Zhang M, *Sharp Upper bounds for Zagreb indices of bipartite graphs with given diameter*, Appl. Math. Lett., 24

- Nikolić S, Kovačević G, Milićević A, Trinajstić N, *The Zagreb indices 30 years after*, Croat, Chem. Acta 76(2003) 113-124.
- Trinajstić N, Nikolić S, Milićević A, Gutman I, On Zagreb indices, Kem.Ind., 59(2010), 577-589.
- Todeschini, R., Ballabio, D., Consonni, V.,*Novel molecular descriptors based on functions of new vertex degrees. In: Novel molecular structure descriptors Theory and applications I.* (I. Gutman, B. Furtula, eds.), 73-100. Kragujevac: Univ. Kragujevac 2010.
- Todeschini R, Consonni V, *New local vertex invariants and molecular descriptors based on functions of the vertex degrees*, MATCH Commun. Math. Comput. Chem. 64 (2010),359372.
- Tutte W.T, *Connectivity in Graphs*, University of Toronto Press/Oxford University Press, London, 1996.

THANK YOU