On Ishikawa Iteration with different control conditions for Asymptotically nonexpansive non-self mappings

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Outline

Abstract

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We prove approximate fixed point property using Ishikawa iteration for asymptotically nonexpansive non-self mappings with different control conditions.

Keywords:

Uniformly convex Banach space, Ishikawa Iterates, asymptotically nonexpansive nonself-maps.

Introduction

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Fixed point theory plays one of the important roles in nonlinear analysis.It is an active area of research with wide range of applications in various directions. It is concerned with the results which state that under certain conditions a self map f on a set X admit one or more fixed points.

Introduction

In 1922, the Polish mathematician Stefan Banach formulated and proved a theorem which concerns under appropriate conditions the existence and uniqueness of a fixed point in a complete metric space.

Introduction

His result is known as Banachs fixed point theorem or the Banachs contraction principle. Due to its simplicity and generality, the contraction principle has drawn attention of a very large number of mathematicians.

Preliminaries

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Definition.

Let E be a real Banach space and let C be a nonempty closed convex subset of E. A map $T : C \to C$ is said to be asymptotically nonexpansive if there exists a sequence$(k_n) \subset [0, 1)$ with $\lim_{n\to\infty} k_n = 1$ such that $\|T^n x - T^n y\| \leq k_n\|x - y\|$ for all $x, y \in C$ and $n \geq 1$.
T is said to be uniformly L-Lipschitzian if $\|T^n x - T^n y\| \leq L\|x - y\|$ for all $x, y \in C$ and $n \geq 1$ where L is a positive constant. For a map T of C into itself, the Ishikawa iteration scheme is studied $x_1 \in C$,
$x_{n+1} = \alpha_n T^n(y_n) + (1 - \alpha_n)x_n$
$y_n = \beta_n T^n(x_n) + (1 - \beta_n)x_n$

Preliminaries

Definition.

Let E be a real normed linear space, K a nonempty subset of E. Let $P : E \to K$ be the nonexpansive retraction of E onto K. A map $T : K \to E$ is said to be asymptotically nonexpansive if there exists a sequence $(k_n) \subset [1, \infty)$, $k_n \to 1 asn \to \infty$ such that the following inequality holds.
$\|T(PT)^{n-1}x - T(PT)^{n-1}y\| \leq k_n\|x - y\|\forall x, y \in K, n \geq 1.$

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T is called uniformly L Lipschitzian if there exists $L > 0$ such that $\|T(PT)^{n-1}x - T(PT)^{n-1}y\| \leq L\|x - y\| \forall x, y \in K, n \geq 1$.

Let K be a nonempty closed convex subset of a real uniformly convex Banach space E. The following iteration scheme is studied

$x_1 \in K, x_{n+1} = P((1 - \alpha_n)x_n + \alpha_n T(PT)^{n-1}y_n),$

$y_n = P((1 - \beta_n)x_n + \beta_n T(PT)^{n-1}x_n),$

Preliminaries

Lemma

Let $r > 0$ be a fixed real number then a Banach space E is uniformly convex if and only if there is a continuous strictly increasing convex map $g : [0, \infty) \to [0, \infty)$ with $g(0) = 0$ such that $\forall x, y \in B_r[0] = \{x \in E : \|x\| \leq r\}$,

$$\|\lambda x + (1-\lambda)y\|^2 \leq \lambda\|x\|^2 + (1-\lambda)\|y\|^2 - \lambda(1-\lambda)g(\|x-y\|) \quad \forall \lambda \in [0, 1].$$

Preliminaries

Lemma

Let $g : [0, \infty) \rightarrow [0, \infty)$ with $g(0) = 0$ be a strictly increasing map. If a sequence $\{x_n\}$ in $[0, \infty)$ satisfies $\lim_{n\to\infty} g(x_n) = 0$, then $\lim_{n\to\infty}(x_n) = 0$.

Main Results

Main Result

Lemma

Let E be a real uniformly convex Banach space, K closed convex nonempty subset of E.
Let $T : K \to E$ be asymptotically nonexpansive with sequence $\{k_n\} \subset [1, \infty)$ such that $\sum_{n\geq1}(k_n - 1) < \infty$ and $F(T) \neq \phi$.
Let $\{\alpha_n\} \subset (0, 1)$ be such that $\epsilon \leq 1 - \alpha_n \leq 1 - \epsilon$ $\forall n \geq 1$ and some $\epsilon > 0$. from arbitrary $x_1 \in K$ define a sequence $\{x_n\}$ then $\lim_{n\to\infty} \|x_n - x^*\|$ exists for each $x^* \in F(T)$.

Main Results

Theorem

Let E be a real uniformly convex Banach space, K closed convex nonempty subset of E. Let $T : K \to E$ be asymptotically nonexpansive with sequence $\{k_n\} \subset [1, \infty)$ such that $\sum_{n\geq 1}(k_n - 1) < \infty$ and $F(T) \neq \phi$. Let $\{\alpha_n\} \subset (0, 1)$ be such that$\epsilon \leq 1 - \alpha_n \leq 1 - \epsilon \forall n \geq 1$ and some $\epsilon > 0$ From arbitrary $x_1 \in k$ define a sequence $\{x_n\}$ by equation . Let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences in $[0, 1]$ and satisfy the following condition :$\sum_{n=1}^{\infty} \alpha_n(1 - \alpha_n) = \infty, \limsup_{n\to\infty} \beta_n < 1$ then

$$\liminf_{n\to\infty} \|x_n - T(PT)^{n-1}x_n\| = 0$$

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