WELCOME

Continuity and Uniform continuity using Epsilon – Delta Property

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Outline

Introduction to Sets

✍ What is set?.

Introduction to Sets

- What is set?.
- Is it merely collection of objects or “things“.?

Introduction to Sets

For Example

The items you wear: shoes, socks, hat, shirt, pants, and so on.

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Types of fingers. This set includes index, middle, ring, and pinky.

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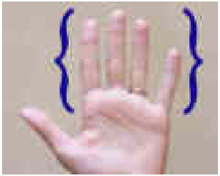

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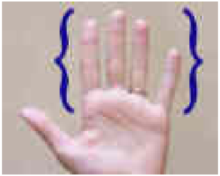

So it is just things grouped together with a certain property in common.

Introduction to Sets

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Well, simply put, it's **a collection**.

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Definition

A set is a collection of well defined objects or things.

Introduction to Sets

Notations

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- Is x is an element of the set S, then it is written as $x \in S$ and read as x belongs to S.
- If x is a not the member of the set S, then it is written as $x \notin S$ and read as x does not belong to S.

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$a \in V$, $i \in V$ but $b \notin V$

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Girls are brilliant.

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Girls are brilliant.

Is it a set ?

No, because here brilliant is not defined.

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$\mathbb{R}$ - Set of Real Numbers $(-\infty, \infty)$

Graphical View

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$\mathbb{N}$

Graphical View

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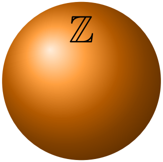
Graphical View



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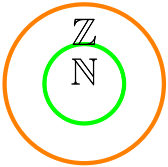
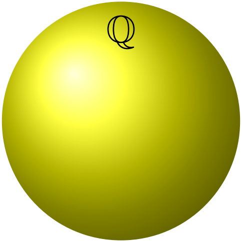


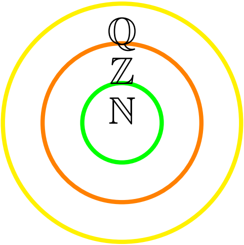
$$\mathbb{N} \subset \mathbb{Z}$$

Graphical View



$\mathbb{N} \subset \mathbb{Z} \quad \mathbb{Q}$

Graphical View



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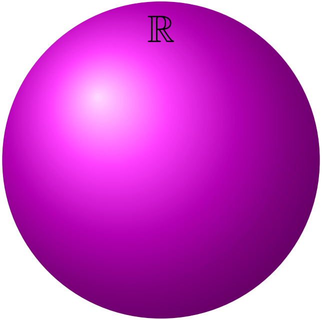
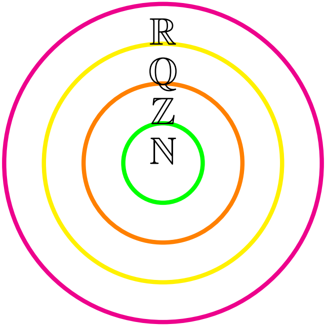


$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \quad \mathbb{R}$

Graphical View



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Function

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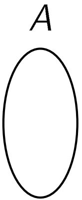
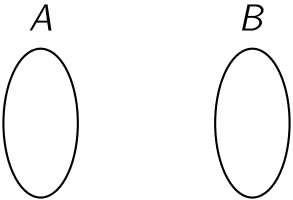
- In mathematically written as $f : A \to B$ defined by $f(a) = b$ for all $a \in A$.



- Consider the function $f : A \rightarrow B$ by $f(a) = b$



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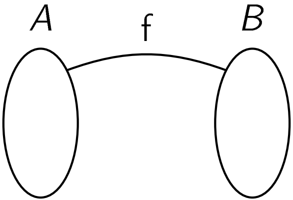

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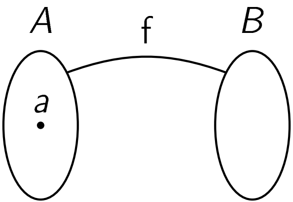

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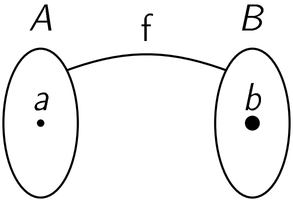

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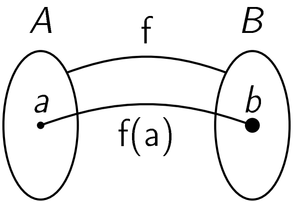

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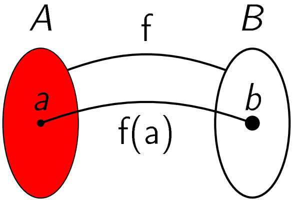
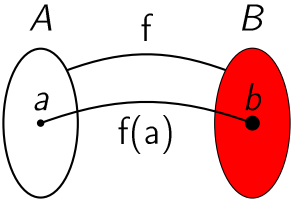


- Consider the function $f : A \to B$ by $f(a) = b$
- A is called the domain of f



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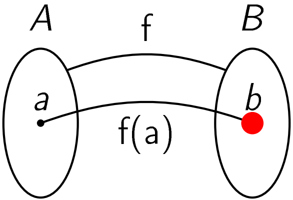

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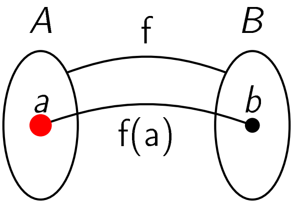

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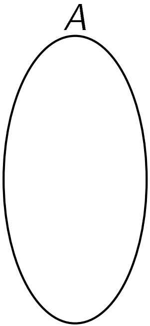
- The element $a \in A$ is called the pre-image of b under f.

Graphical



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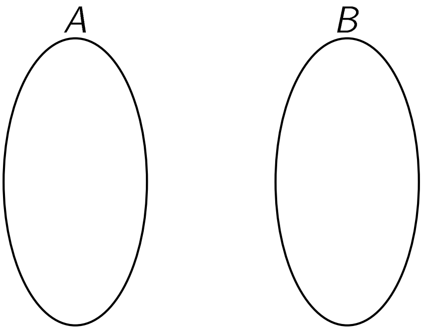

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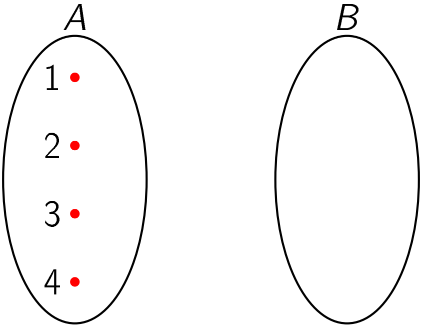

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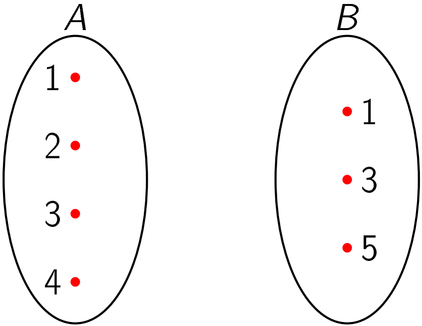

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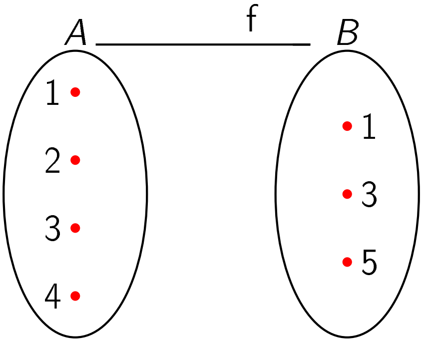

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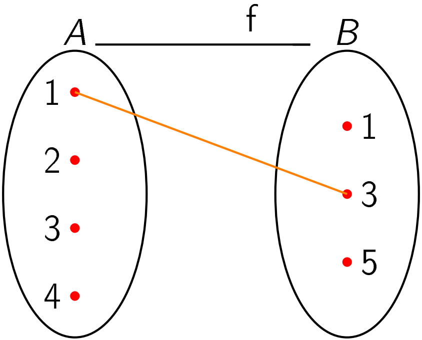

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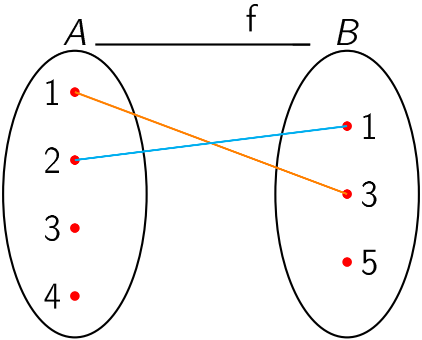

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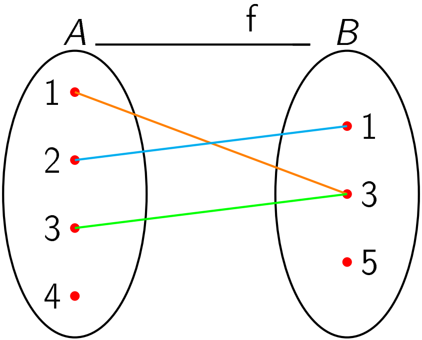

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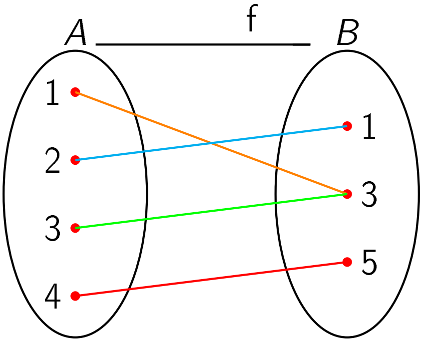

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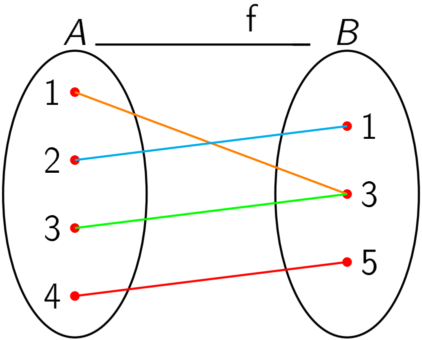
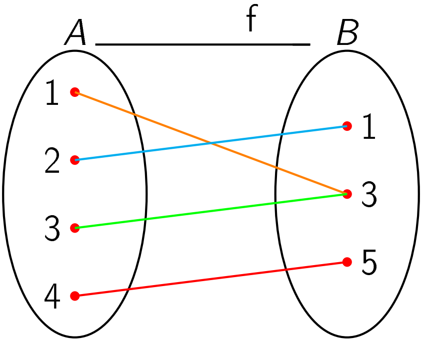


Is it function ?

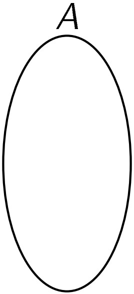
Graphical



Is it function ?

Yes

Graphical



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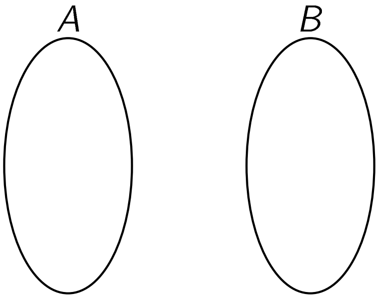

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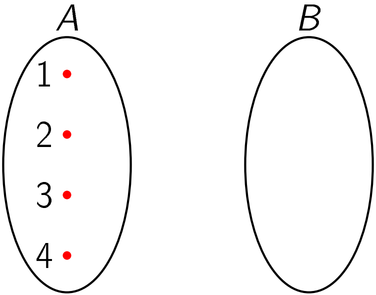

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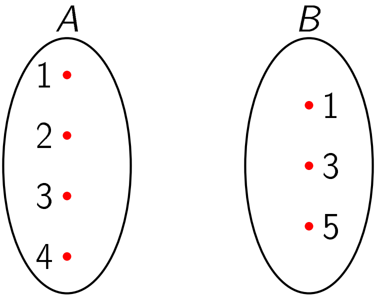

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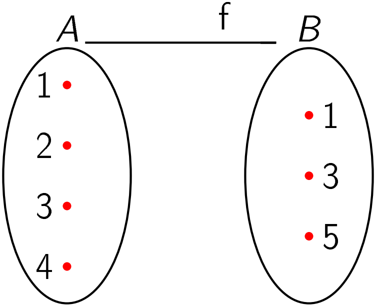

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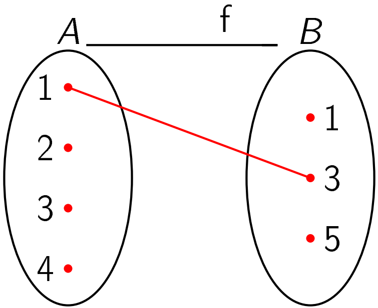

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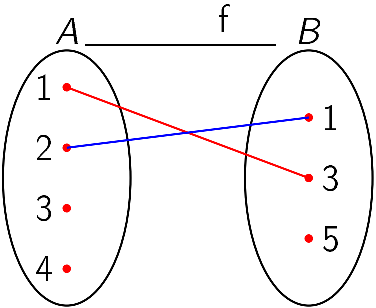

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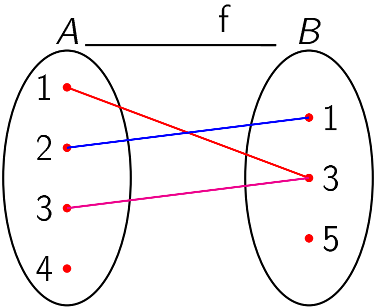

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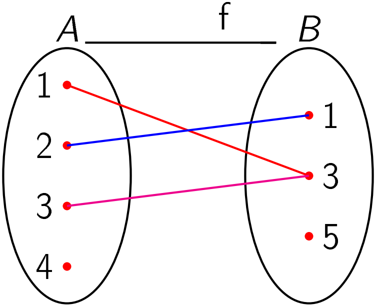

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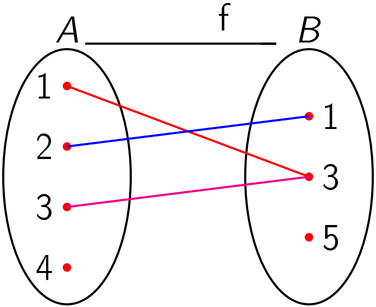
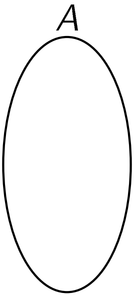
Graphical



Is it function ?

No

Graphical



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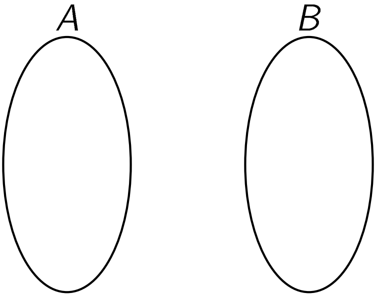

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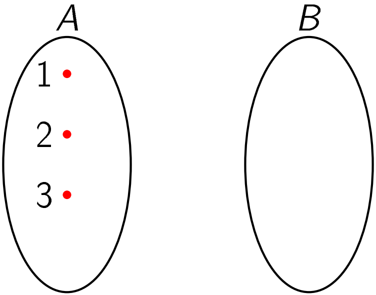

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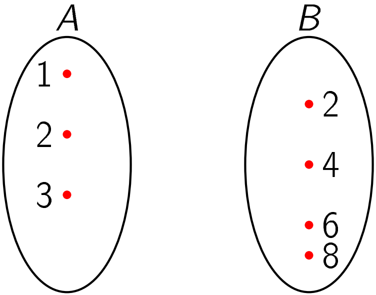

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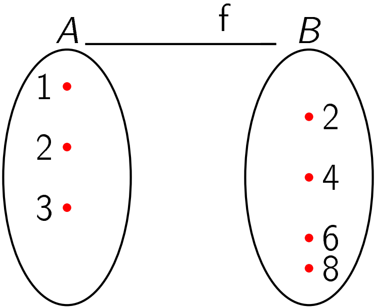

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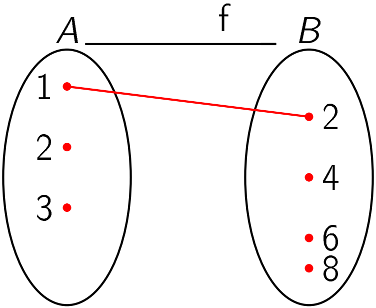

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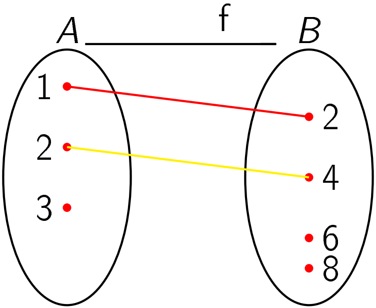

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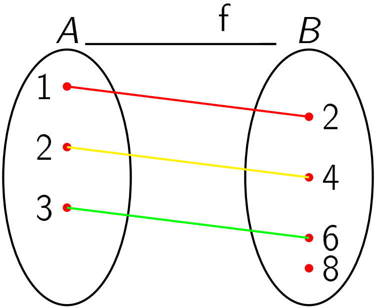

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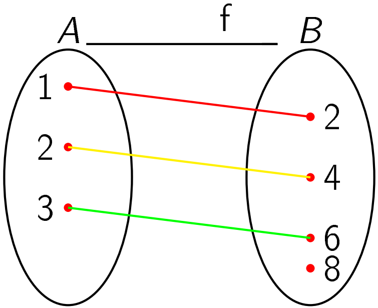
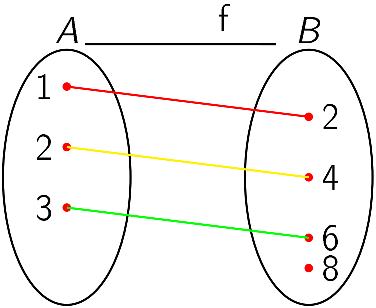


Is it function ? If it is, what type is it?

Graphical



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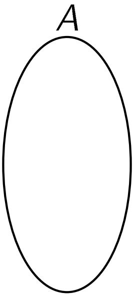
One - to - one (or) Injective

Graphical



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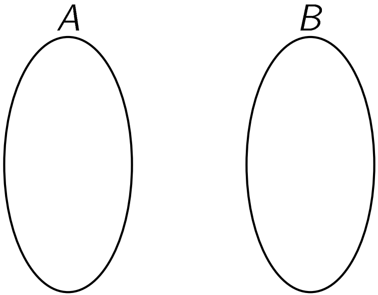

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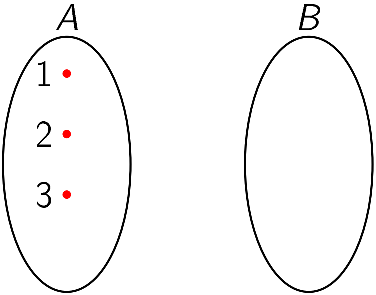

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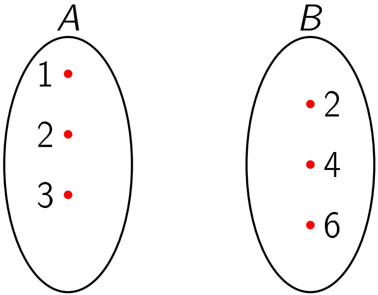

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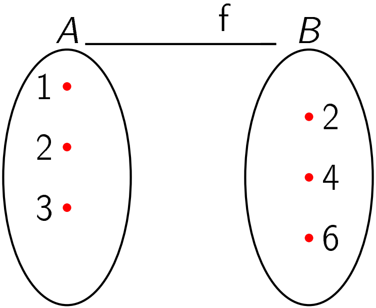

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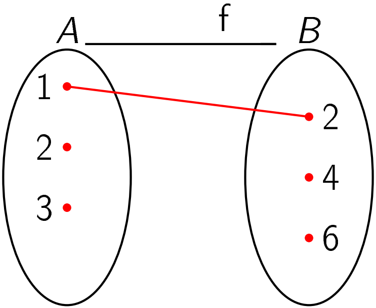

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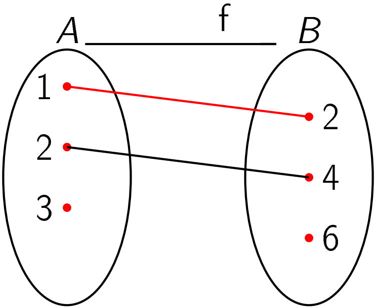
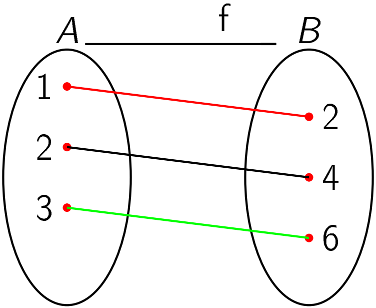


Graphical



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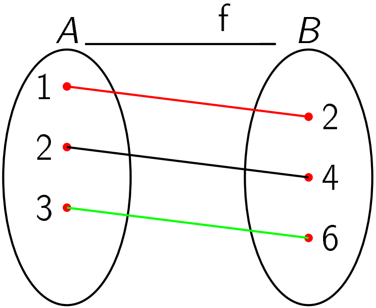
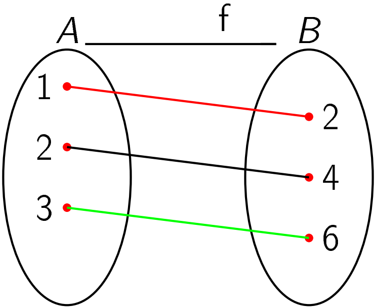


Is it function ? If it is, what type is it?

Graphical



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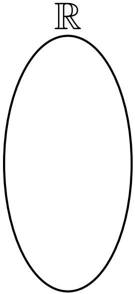
Onto (or) Surjective

Graphical



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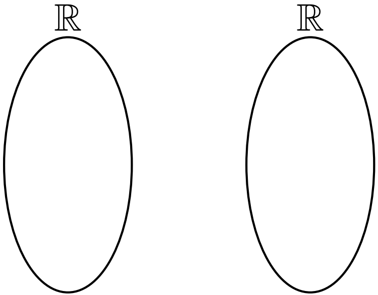

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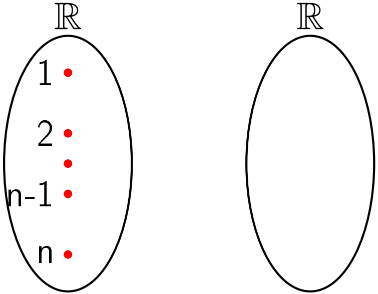

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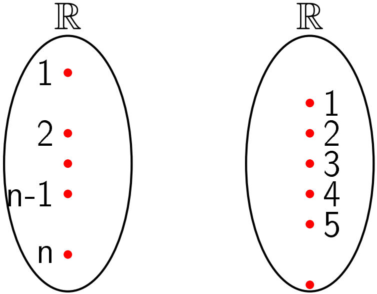

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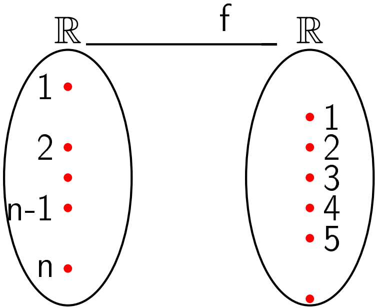

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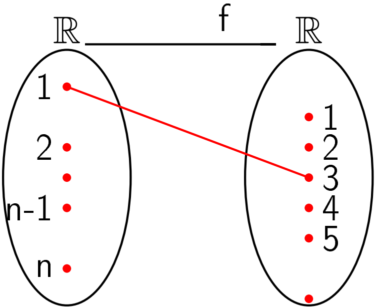

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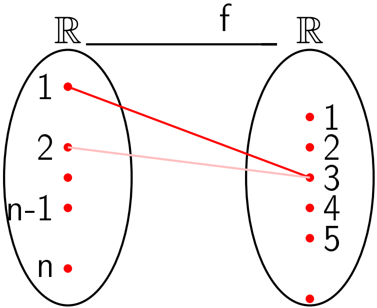

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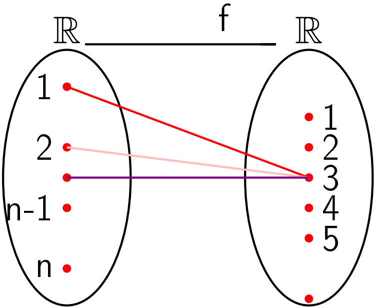

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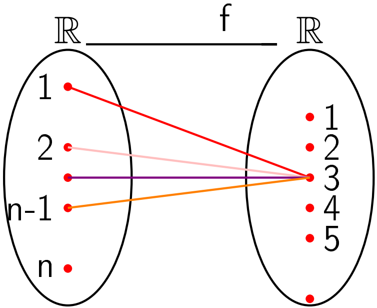

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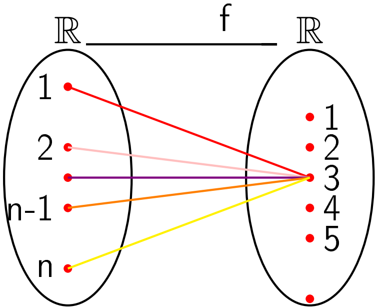

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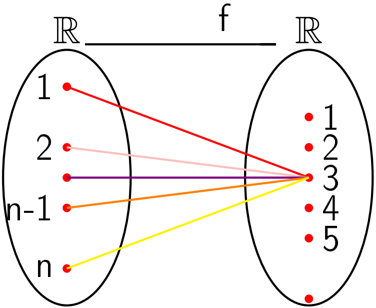

Constant Function

$f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 3$ is called a constant function. The range of f is 3.

Introducing Sequence

☞ In maths, we call a list of numbers in order a sequence.

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- ☞ When we write out sequences, consecutive terms are usually separated by commas.

Example

Consider the following collection of real numbers given by

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$$

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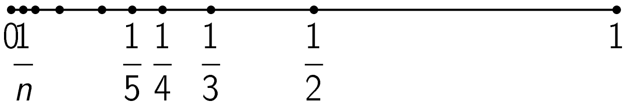

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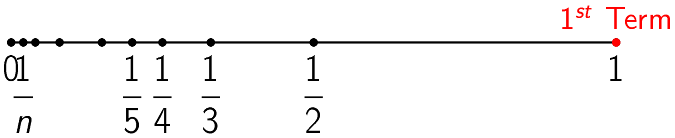

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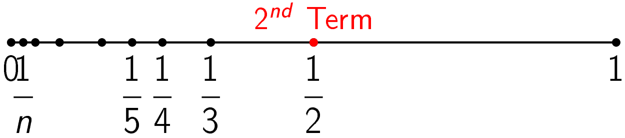

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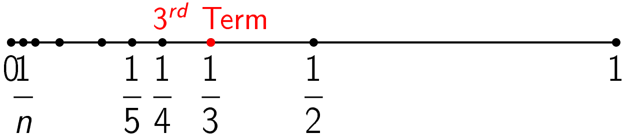

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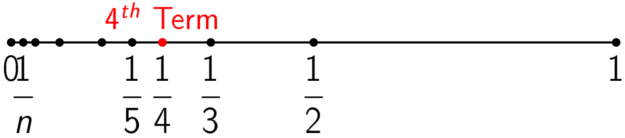

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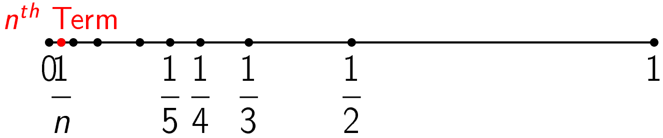

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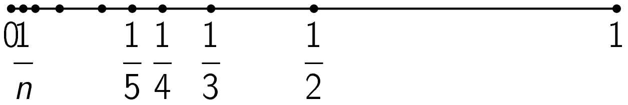

This is an example of sequence of real numbers.

Sequence is a function whose domain is the set of natural numbers.

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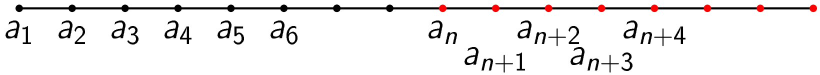
Definition

Let $f : \mathbb{N} \rightarrow \mathbb{R}$ be a function and $f(n) = a_n$. Then $a_1, a_2, a_3, \cdots, a_n, \cdots$, is called the sequence in $\mathbb{R}$ determined by the function f and is denoted by $\{a_n\}$, a_n is called the n^{th} term of the sequence.

Convergence of a Sequence

We say that a sequence (x_n) converges if there exists $x_0 \in \mathbb{R}$ such that for every $\epsilon > 0$, there exists a positive integer N (depending on ϵ) such that $x_n \in (x_0 - \epsilon, x_0 + \epsilon)$ for all $n \geq N$.



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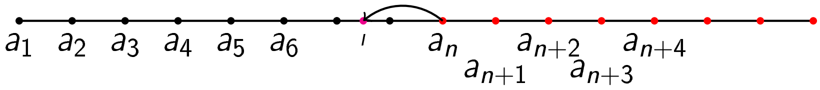

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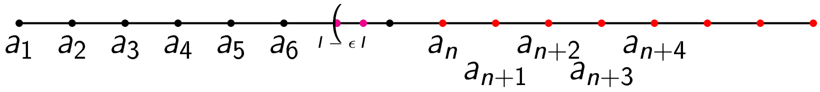
Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$



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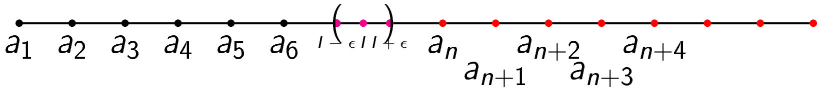

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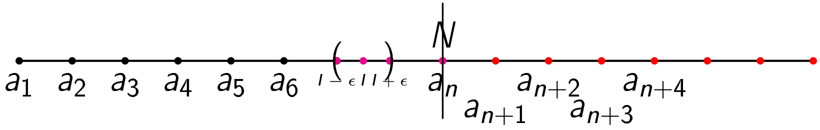

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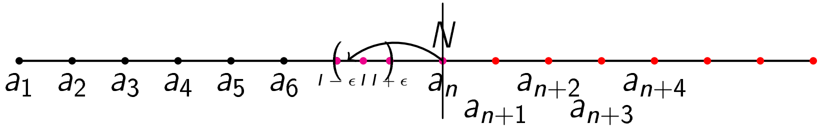

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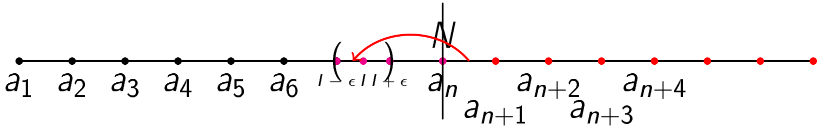

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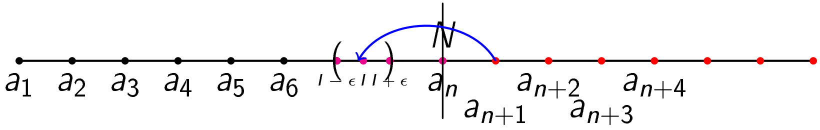

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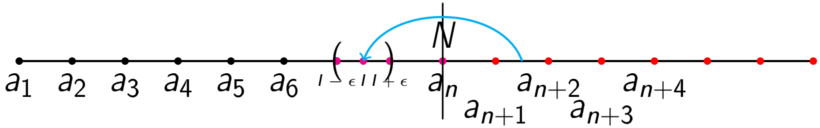

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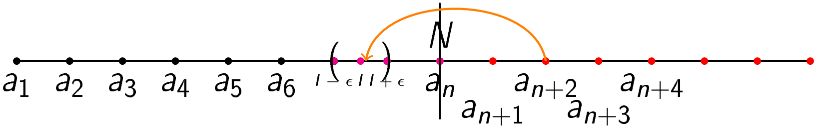

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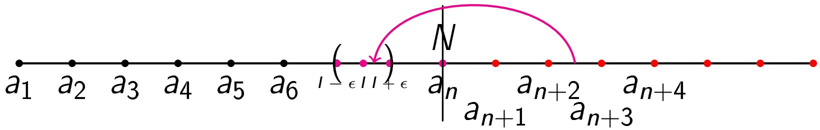

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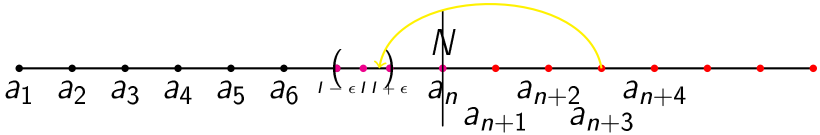

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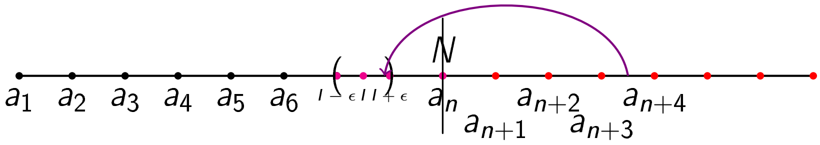

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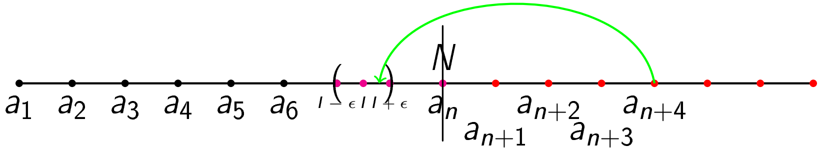

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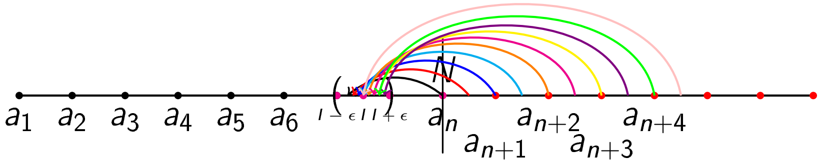

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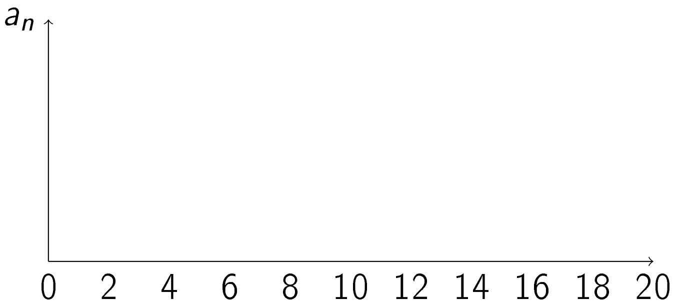
a_n
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2
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6
8
10
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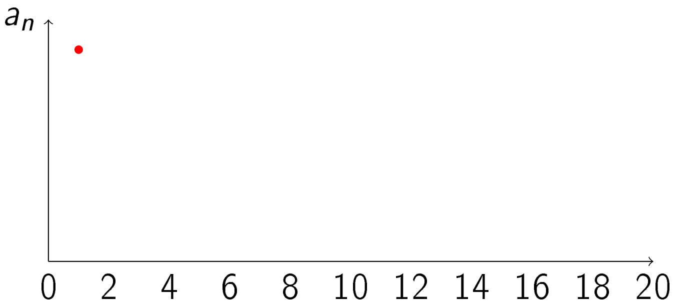
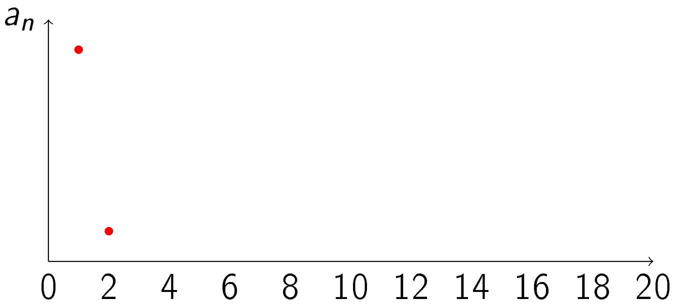
16
18
20


a_n
0 2 4 6 8 10 12 14 16 18 20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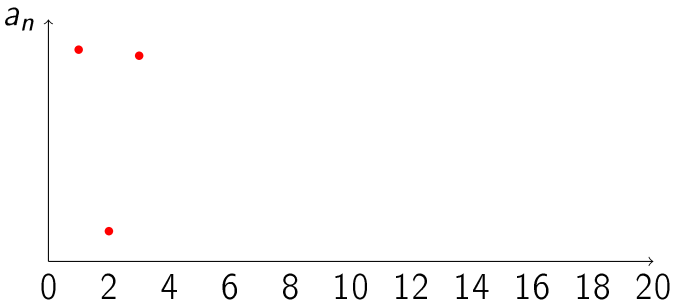
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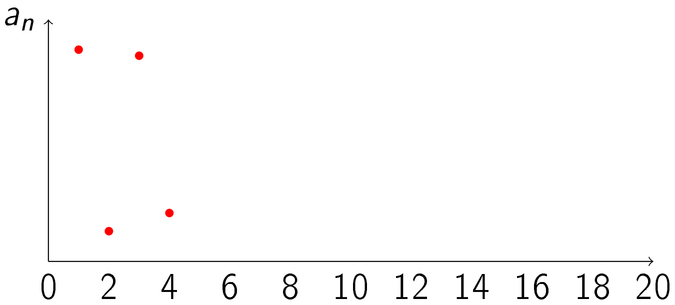
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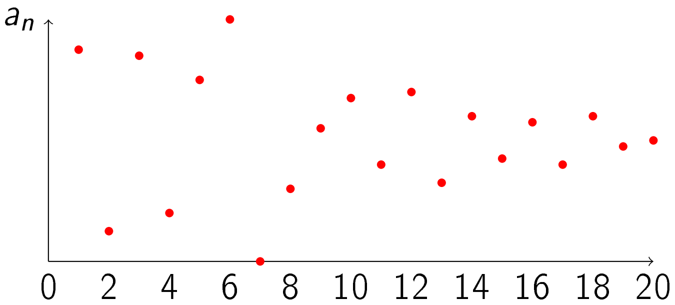
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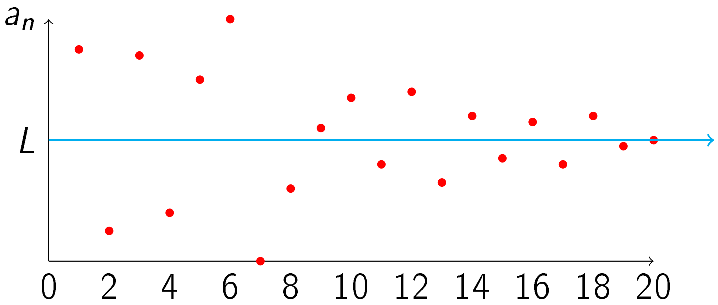
a_n
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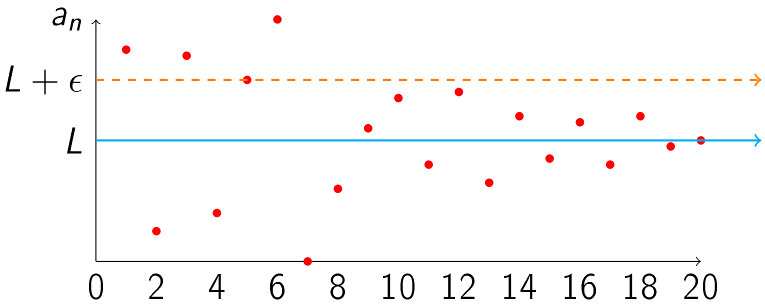
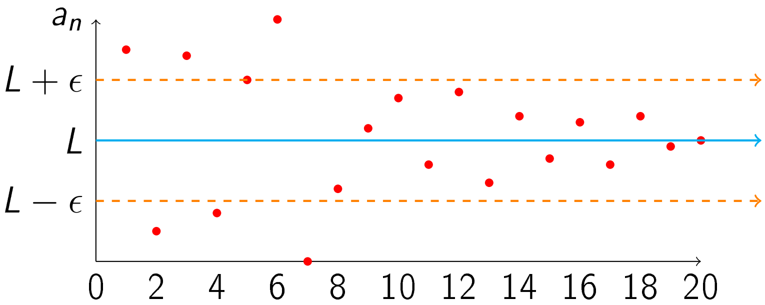
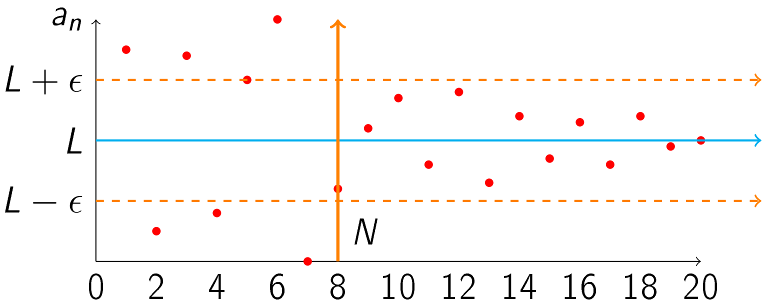
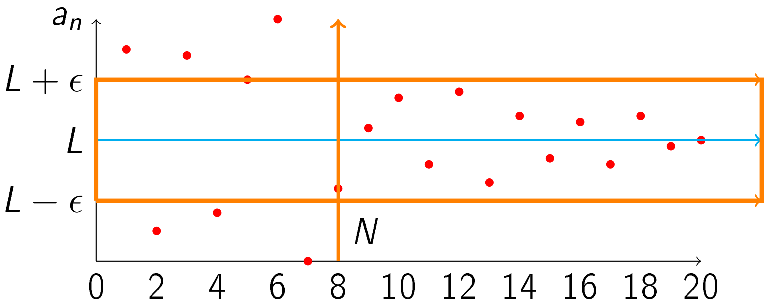

a_n
$L+\epsilon$
L
0 2 4 6 8 10 12 14 16 18 20


a_n
$L+\epsilon$
L
$L-\epsilon$
0 2 4 6 8 10 12 14 16 18 20


a_n
$L + \epsilon$
L
$L - \epsilon$
N
0 2 4 6 8 10 12 14 16 18 20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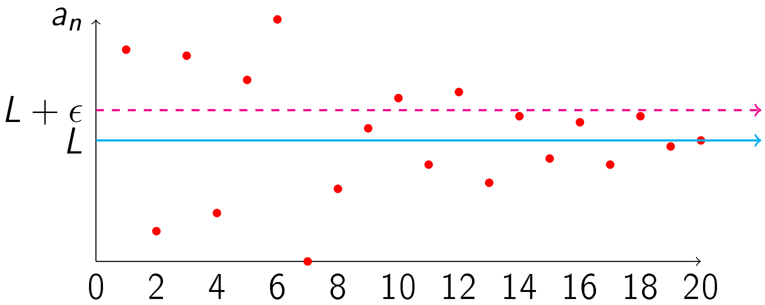
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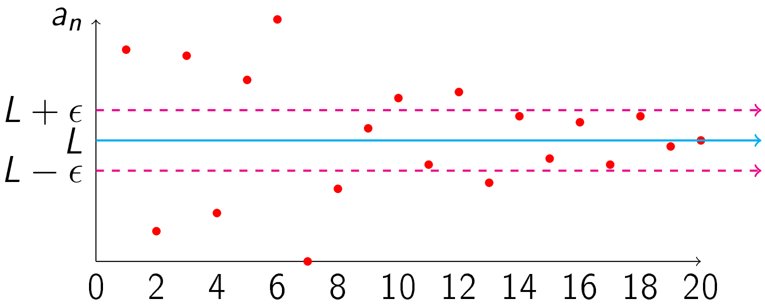

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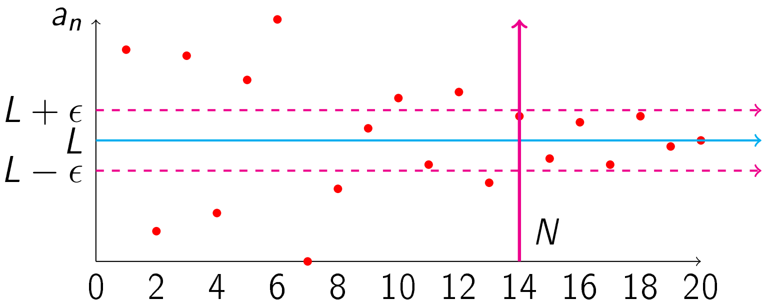

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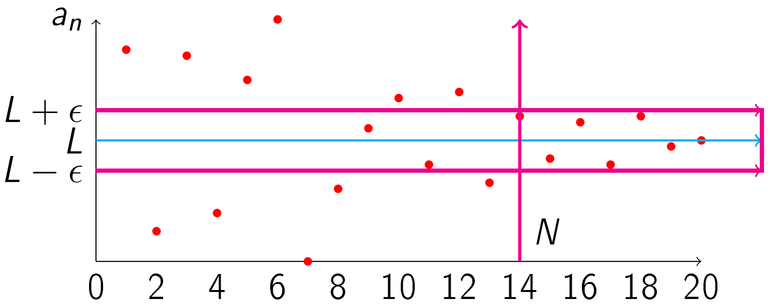
a_n
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$L - \epsilon$
N
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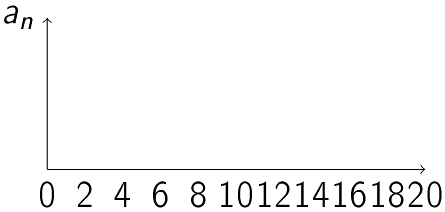
Graphical View



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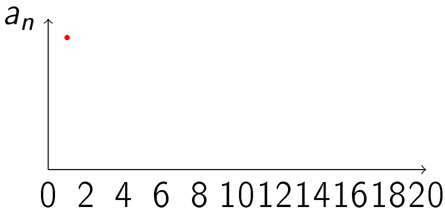

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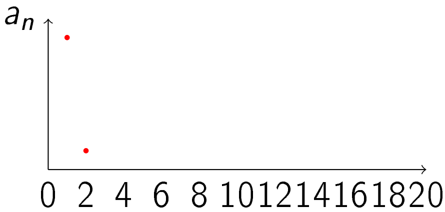

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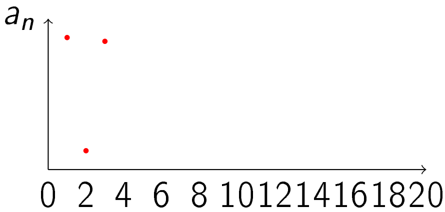

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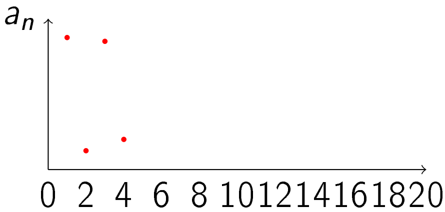
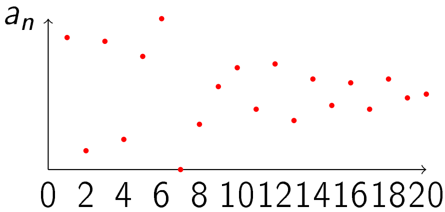


Graphical View



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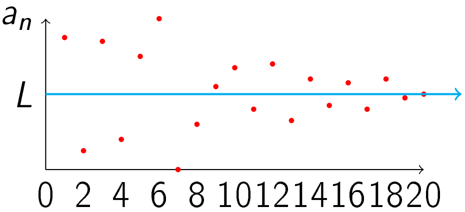

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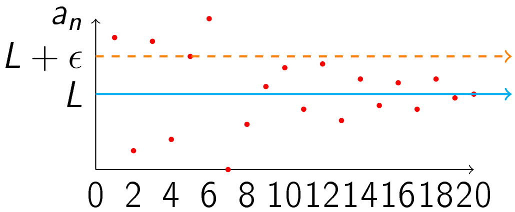

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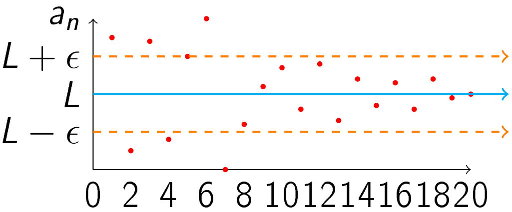

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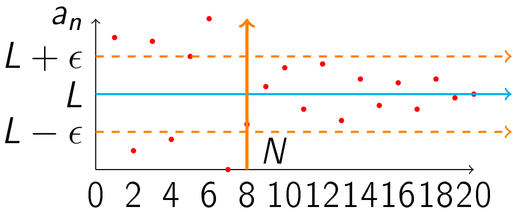

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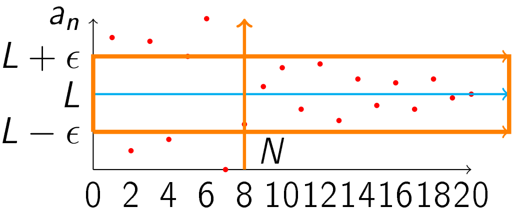
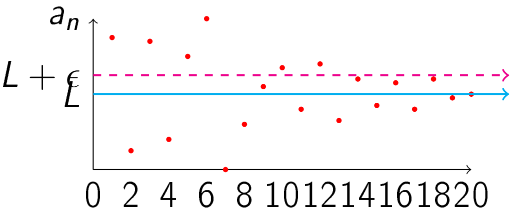
Graphical View



For any $\epsilon > 0$, $\exists$ a positive integer N such that $|a_n - L| \leq \epsilon$ for all $n > m$.

Graphical View



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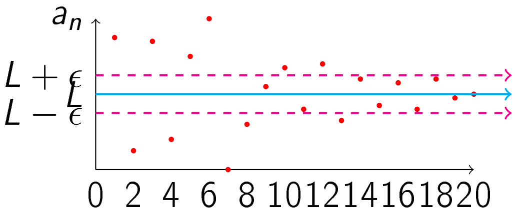

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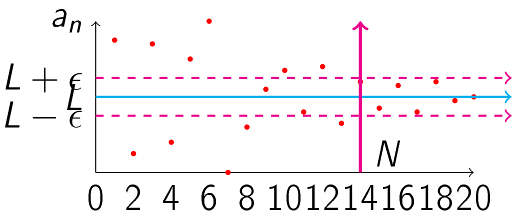

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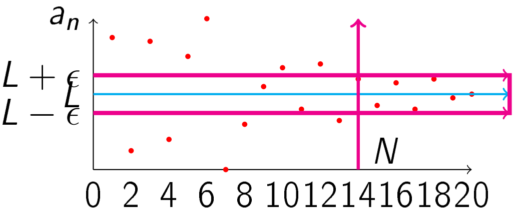

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Properties of sequence

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4. Any convergent sequence is bounded.

Concept

Continuous functions are functions that take nearby values at nearby points.

Origin

- The term continuous has been used since the time of Newton to refer to the motion of bodies or to describe an unbroken curve

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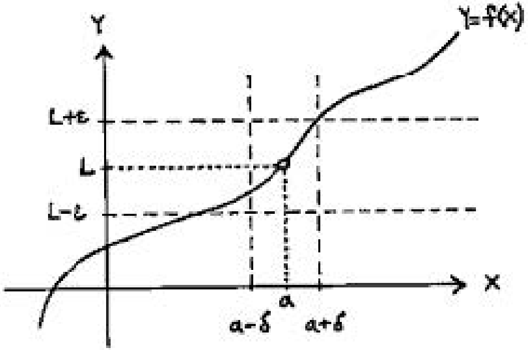
- The term continuous has been used since the time of Newton to refer to the motion of bodies or to describe an unbroken curve
- It was made precise until the Nineteenth century.
- Work of Bernard Bolzano in 1817 and Cauchy 1821 identified continuity as a very significant property of function
- The concept is tied to that of limit, it was the careful work of Weierstrassin the 1870s that brought proper understanding to the idea of continuity.

Continuous function

Let $f : A \longrightarrow R$, where $A \subset R$, and suppose that $c \in A$. Then f is continuous at c if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $|x - c| < \delta$ and $x \in A$ implies that $|f(x) - f(c)| < \varepsilon$.

Graph



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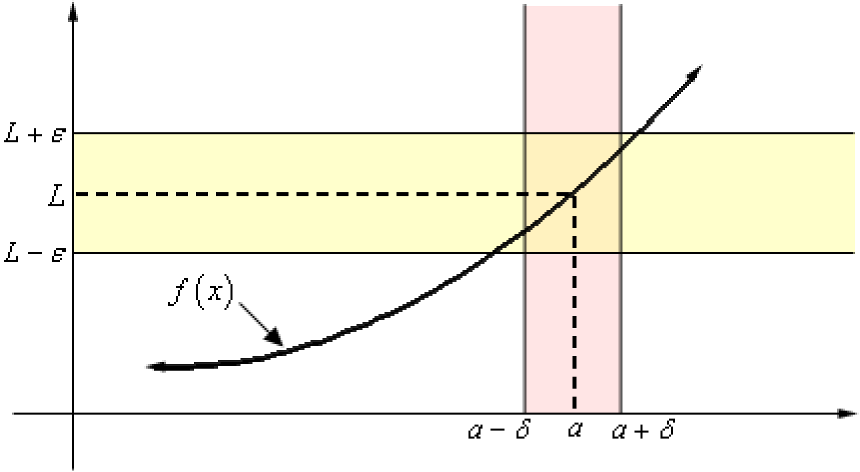

Note

A function $f : A \longrightarrow R$ is continuous on a set $B \subset A$ if it is continuous at every point in B, and continuous if it is continuous at every point of its domain.

Steps

1. Take $|f(x) - f(c)| < \varepsilon$ and rewrite it to match $|x - c| < \delta$ to create a direct relationship

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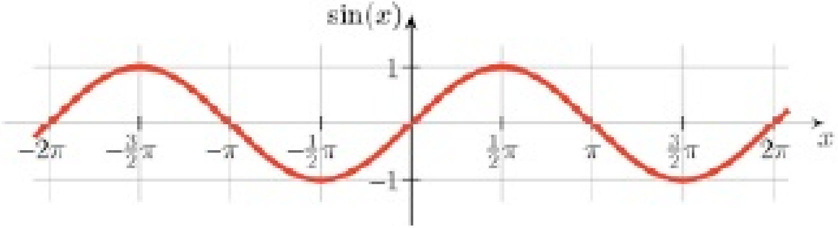
1. Take $|f(x) - f(c)| < \varepsilon$ and rewrite it to match $|x - c| < \delta$ to create a direct relationship
2. Let $|x - c| < \delta$ and prove $|f(x) - f(c)| < \varepsilon$

Continuous function

The function $sinx : R \longrightarrow R$is continuous on R.

Sinx curve



Continuous function

Choose $\delta = \varepsilon$ in the definition of continuity for every $c \in R$

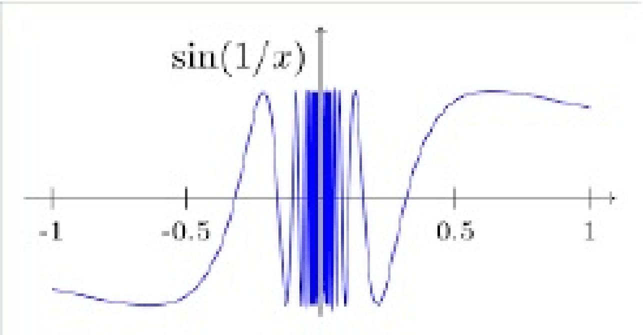
Continuous function

The function $f : R \longrightarrow R$ defined by

$$f(x) = \begin{cases} \sin(1/x) & , \text{ if } x \neq 0, \\ 0 & , \text{ if } x = 0 \end{cases}$$

is continuous on $R - 0$, since it is the composition of $x \mapsto 1/x$, which is continuous on $R - 0$ and $y \mapsto siny$, which is continuous on R.

$Sin(\frac{1}{x})$ curve



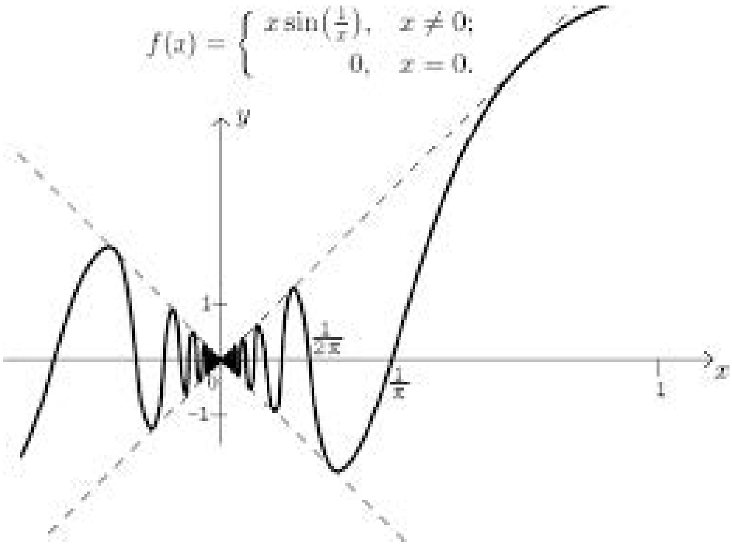
Continuous function

The function $f : R \longrightarrow R$ defined by

$$f(x) = \begin{cases} xsin(1/x) & \text{if} x \neq 0, \\ 0 & \text{if} x = 0. \end{cases}$$

Then f is continuous at 0.

$xSin(\frac{1}{x})$ curve



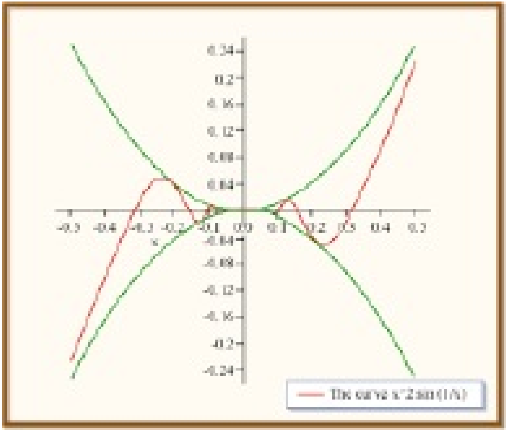
Continuous function

The function $f : R \longrightarrow R$ defined by

$$f(x) = \begin{cases} x^2 sin(1/x) & \text{if} x \neq 0, \\ 0 & \text{if} x = 0. \end{cases}$$

is continuous at 0.

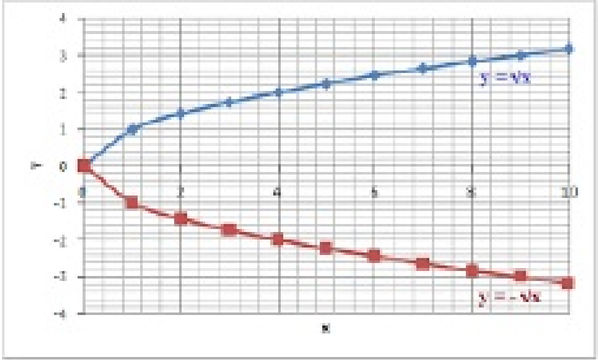
$x^2 Sin(\frac{1}{x})$ curve



Continuous function

The function $f : [0, \infty) \longrightarrow R$ defined by $f(x) = \sqrt{x}$ is continuous on $[0, \infty)$. (i) Prove that f is continuous at c > 0, we can choose $\delta = \sqrt{c}\varepsilon > 0$
(ii) Prove that f is continuous at 0, we note that if $0 \leq x < \delta$ where $\delta = \varepsilon^2 > 0$,

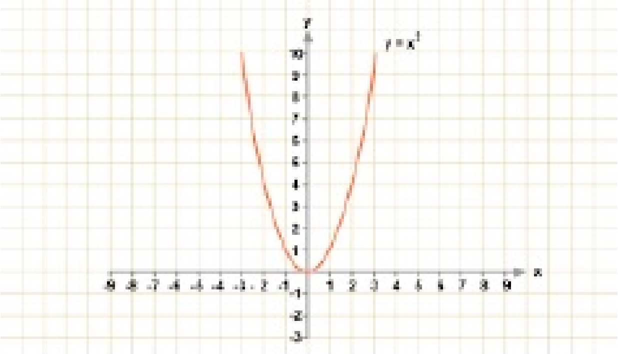
$$f(x) = \sqrt{x} \text{ curve}$$



Continuous function

The function $f(x) = x^2 + 1$ is continuous at $x = 2$

x^2 curve



Uniform Continuous function

Let $f : A \longrightarrow R$, where $A \subset R$. Then f is uniformly continuous on A if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $|x - y| <$ and $x, y \in A$ implies that $|f(x) - f(y)| < \varepsilon$.

Remarks

- The key point of this definition is that δ depends only on ε, not on x, y.

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- The key point of this definition is that δ depends only on ε, not on x, y.
- A uniformly continuous function on A is continuous at every point of A, but the converse is not true.

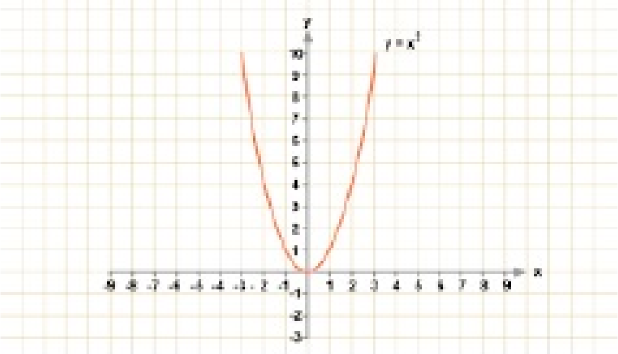
Continuous function

The sine function is uniformly continuous on R, since we can take $\delta = \varepsilon$ for every $x, y \in R$.

Continuous function

Define $f : [0, 1] \longrightarrow R$ by $f(x) = x^2$. Then f is uniformly continuous on $[0, 1]$.

x^2 curve



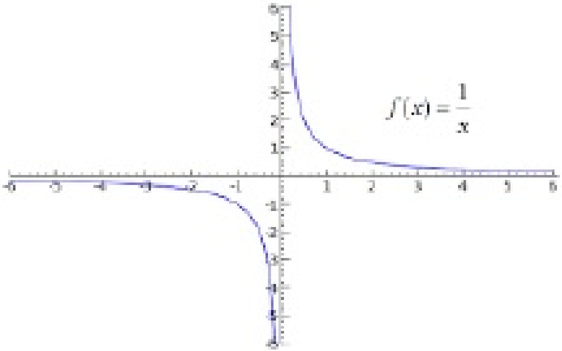
Continuous function but not uniform

The function $f(x) = x^2$ is continuous but not uniformly continuous on R.

Continuous function

The function $f : (0,1] \longrightarrow R$ defined by $f(x) = \frac{1}{x}$
is continuous but not uniformly continuous on $(0,1]$.

$\left(\frac{1}{x}\right)$ curve



Continuous function but not uniform

Define $f : (0, 1] \longrightarrow R$ by $f(x) = sin(\frac{1}{x})$
Then f is continuous on $(0, 1]$ but it is not uniformly continuous on $(0, 1]$.

Time to Interact

Thank You