WELCOME

Continuity and Uniform continuity using Epsilon – Delta Property

J.Maria Joseph PhD

Assistant Professor,
P.G. and Research Department of Mathematics,
St.Joseph's College (Autonomous),
Tiruchirappalli - 620 002, India.

St.Joseph's College,Trichy

Outline

Introduction to Sets

✍ What is set?.

Introduction to Sets

- ✍ What is set?.
- ✍ Is it merely collection of objects or “things“.?

Introduction to Sets

For Example

The items you wear: shoes, socks, hat, shirt, pants, and so on.

Introduction to Sets

For Example

The items you wear: shoes, socks, hat, shirt, pants, and so on. I'm sure you could come up with at least a hundred.

Introduction to Sets

For Example

The items you wear: shoes, socks, hat, shirt, pants, and so on. I'm sure you could come up with at least a hundred. This is known as a **set**.

Introduction to Sets

For Example

The items you wear: shoes, socks, hat, shirt, pants, and so on. I'm sure you could come up with at least a hundred. This is known as a **set**.

Introduction to Sets

For Example

Types of fingers.

Introduction to Sets

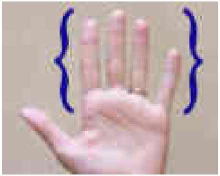
For Example

Types of fingers. This set includes index, middle, ring, and pinky.

Introduction to Sets

For Example

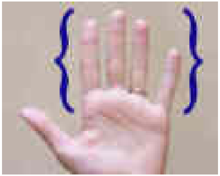
Types of fingers. This set includes index, middle, ring, and pinky.



Introduction to Sets

For Example

Types of fingers. This set includes index, middle, ring, and pinky.



So it is just things grouped together with a certain property in common.

Introduction to Sets

What is set ?

Well, simply put, it's **a collection**.

Introduction to Sets

What is set ?

Well, simply put, it's **a collection**.

Definition

A set is a collection of well defined objects or things.

Introduction to Sets

Notations

- Sets are generally denoted by capital letters $A, B, C, \cdots$ etc.,

Introduction to Sets

Notations

- Sets are generally denoted by capital letters $A, B, C, \cdots$ etc.,
- Elements of the sets are denoted by the small letters $a, b, c, d, e, f, \cdots$ etc.,

Introduction to Sets

Notations

- Sets are generally denoted by capital letters $A, B, C, \cdots$ etc.,
- Elements of the sets are denoted by the small letters $a, b, c, d, e, f, \cdots$ etc.,
- Is x is an element of the set S, then it is written as $x \in S$ and read as x belongs to S.

Introduction to Sets

Notations

- Sets are generally denoted by capital letters $A, B, C, \cdots$ etc.,
- Elements of the sets are denoted by the small letters $a, b, c, d, e, f, \cdots$ etc.,
- Is x is an element of the set S, then it is written as $x \in S$ and read as x belongs to S.
- If x is a not the member of the set S, then it is written as $x \notin S$ and read as x does not belong to S.

Introduction to Sets

Example

Consider the set $V = \{a, e, i, o, u\}$

$a \in V$, $i \in V$ but $b \notin V$

Introduction to Sets

Example

Consider the set $V = \{a, e, i, o, u\}$

$a \in V$, $i \in V$ but $b \notin V$

V is the set of vowels in alphabet.

Introduction to Sets

Example

Consider the set $V = \{a, e, i, o, u\}$
$a \in V$, $i \in V$ but $b \notin V$
V is the set of vowels in alphabet.

Is it ?

Girls are brilliant.

Introduction to Sets

Example

Consider the set $V = \{a, e, i, o, u\}$
$a \in V$, $i \in V$ but $b \notin V$
V is the set of vowels in alphabet.

Is it ?

Girls are brilliant.
Is it a set ?

Introduction to Sets

Example

Consider the set $V = \{a, e, i, o, u\}$
$a \in V$, $i \in V$ but $b \notin V$
V is the set of vowels in alphabet.

Is it ?

Girls are brilliant.
Is it a set ?
No, because here brilliant is not defined.

Sets

$\mathbb{N}$ - Natural Numbers $\{1, 2, 3, 4, \cdots\}$

Sets

$\mathbb{N}$ - Natural Numbers $\{1, 2, 3, 4, \cdots\}$
$\mathbb{Z}$ - Set of Integers $\{0, \pm1, \pm2, \pm3, \pm4, \cdots\}$

Sets

$\mathbb{N}$ - Natural Numbers $\{1, 2, 3, 4, \cdots\}$

$\mathbb{Z}$ - Set of Integers $\{0, \pm1, \pm2, \pm3, \pm4, \cdots\}$

$\mathbb{Q}$ - Set of Rational Numbers $\{\frac{p}{q},\ q \neq 0\}$

Sets

$\mathbb{N}$ - Natural Numbers $\{1, 2, 3, 4, \cdots\}$

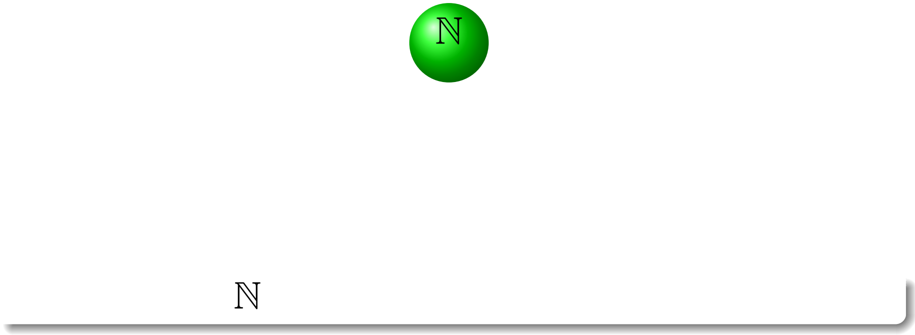
$\mathbb{Z}$ - Set of Integers $\{0, \pm 1, \pm 2, \pm 3, \pm 4, \cdots\}$

$\mathbb{Q}$ - Set of Rational Numbers $\{\frac{p}{q}, \ q \neq 0\}$

$\mathbb{R}$ - Set of Real Numbers $(-\infty, \infty)$

Graphical View

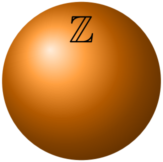
Graphical View



Graphical View

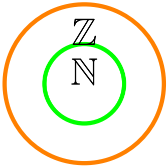
$\mathbb{N}$

Graphical View



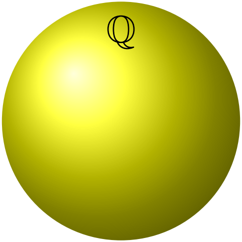
$\mathbb{N}$ $\mathbb{Z}$

Graphical View



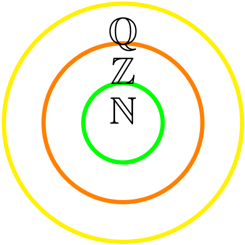
$\mathbb{N} \subset \mathbb{Z}$

Graphical View



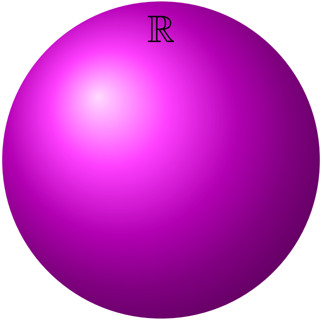
$\mathbb{N} \subset \mathbb{Z} \quad \mathbb{Q}$

Graphical View



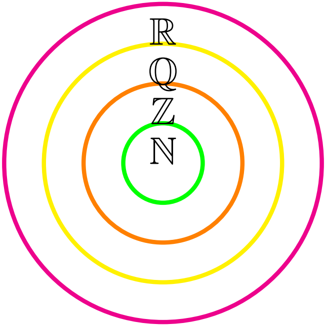
$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$$

Graphical View



$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \quad \mathbb{R}$

Graphical View



$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

Function

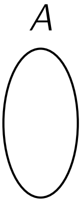
- Function - Relation between two non-empty sets.

Function

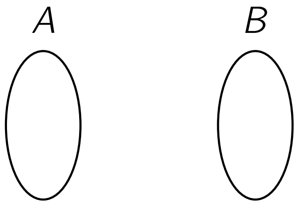
- Function - Relation between two non-empty sets.
- Let A and B be two non-empty sets. A function or mapping f from A into B is a rule which assigns each element $a \in A$ a unique element $b \in B$.

Function

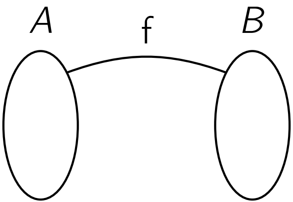
- Function - Relation between two non-empty sets.
- Let A and B be two non-empty sets. A function or mapping f from A into B is a rule which assigns each element $a \in A$ a unique element $b \in B$.
- In mathematically written as $f : A \to B$ defined by $f(a) = b$ for all $a \in A$.



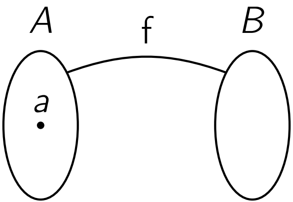
- Consider the function $f : A \to B$ by $f(a) = b$



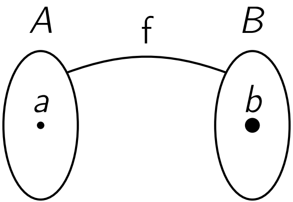
- Consider the function $f : A \to B$ by $f(a) = b$



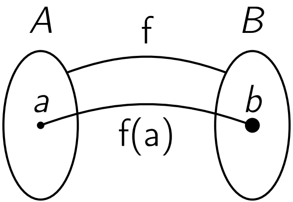
- Consider the function $f : A \to B$ by $f(a) = b$



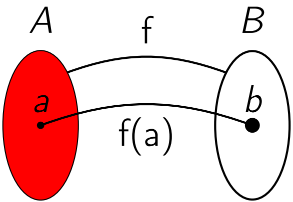
- Consider the function $f : A \to B$ by $f(a) = b$



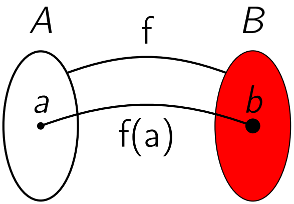
Consider the function $f : A \to B$ by $f(a) = b$



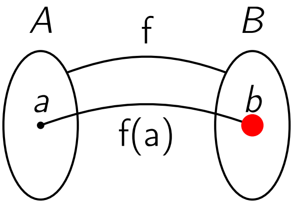
✤ Consider the function $f : A \to B$ by $f(a) = b$



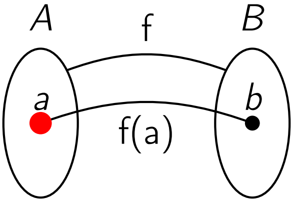
- Consider the function $f : A \to B$ by $f(a) = b$
- A is called the domain of f



- Consider the function $f : A \to B$ by $f(a) = b$
- A is called the domain of f
- B is called the co-domain of f

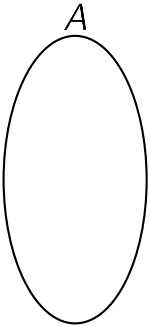


- Consider the function $f : A \to B$ by $f(a) = b$
- A is called the domain of f
- B is called the co-domain of f
- The element $b \in B$ is called the image of a under f.

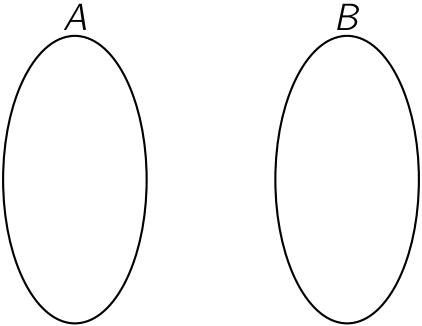


- Consider the function $f : A \to B$ by $f(a) = b$
- A is called the domain of f
- B is called the co-domain of f
- The element $b \in B$ is called the image of a under f.
- The element $a \in A$ is called the pre-image of b under f.

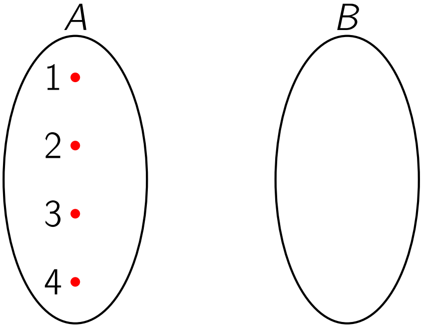
Graphical



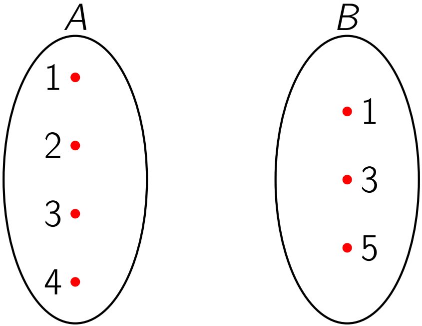
Graphical



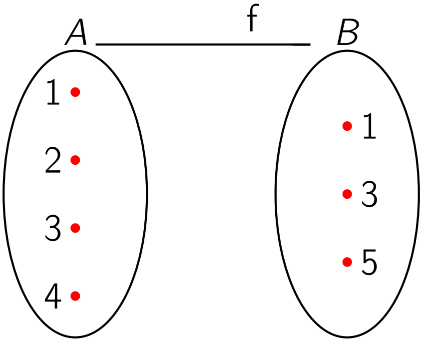
Graphical



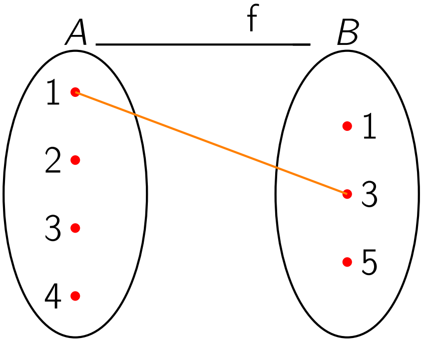
Graphical



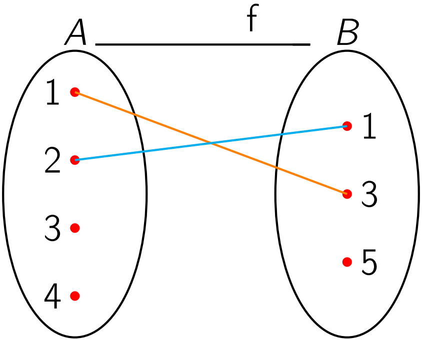
Graphical



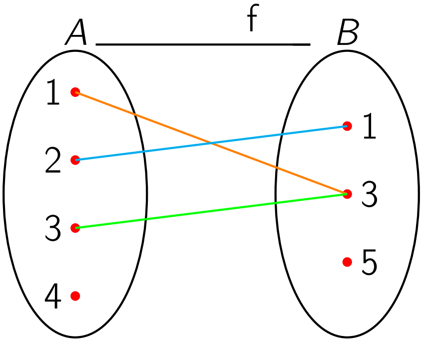
Graphical



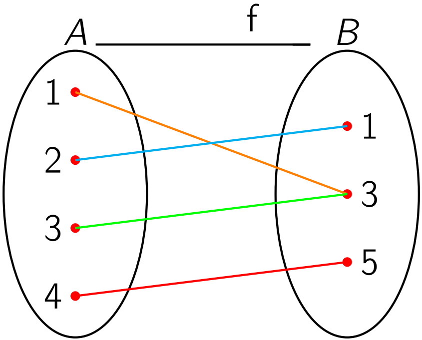
Graphical



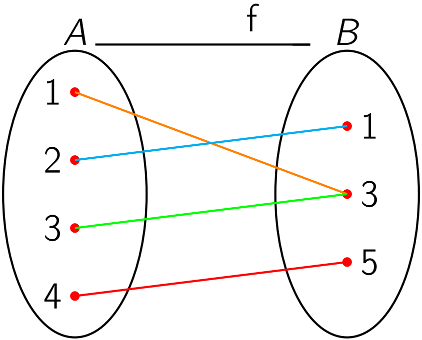
Graphical



Graphical

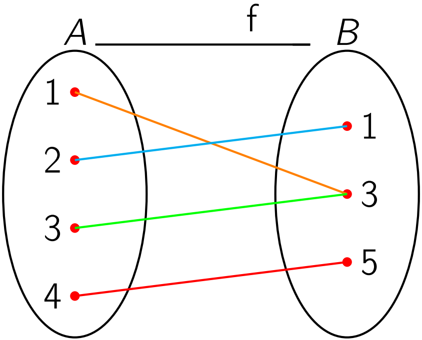


Graphical



Is it function ?

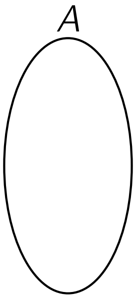
Graphical



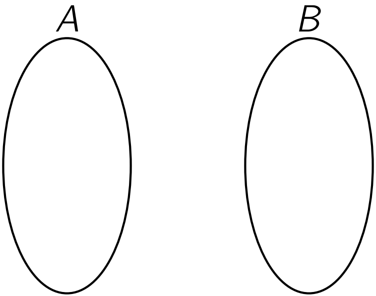
Is it function ?

Yes

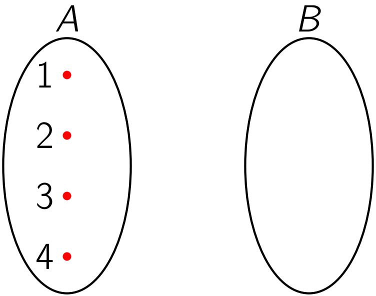
Graphical



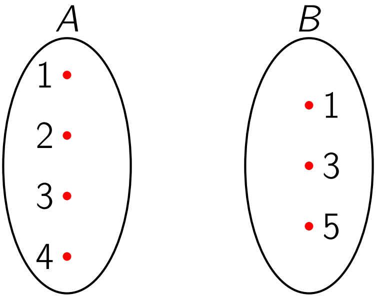
Graphical



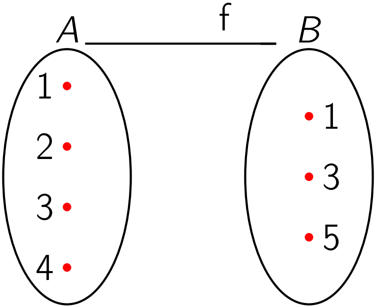
Graphical



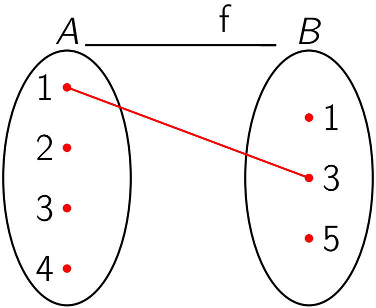
Graphical



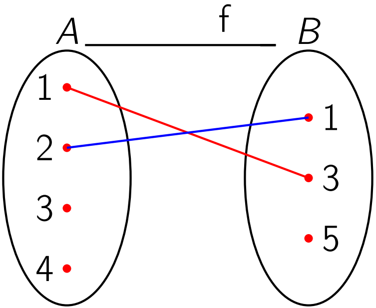
Graphical



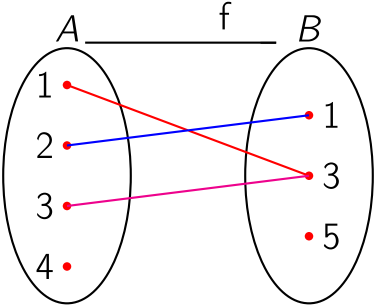
Graphical



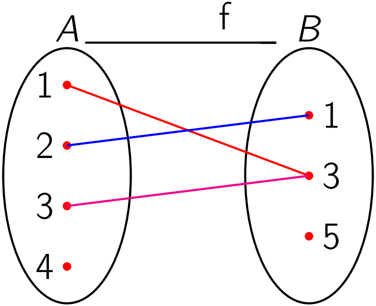
Graphical



Graphical

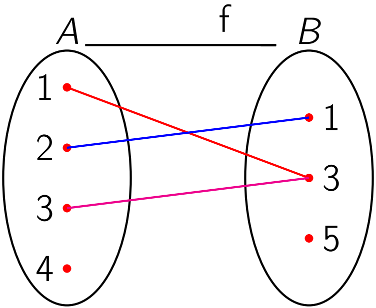


Graphical



Is it function ?

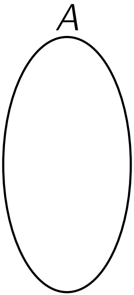
Graphical



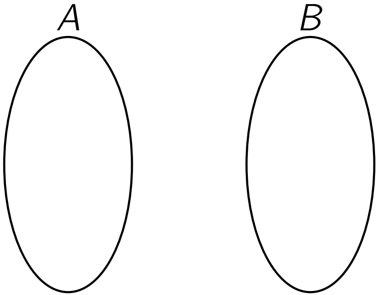
Is it function ?

No

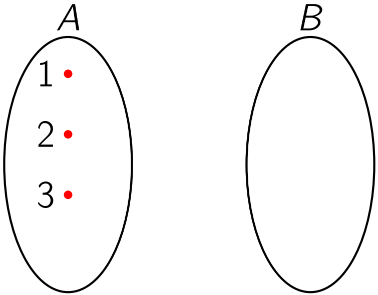
Graphical



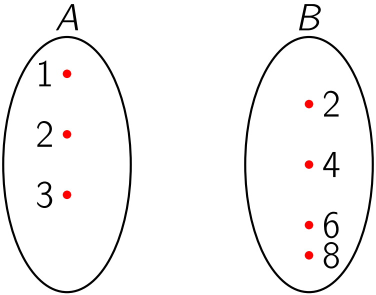
Graphical



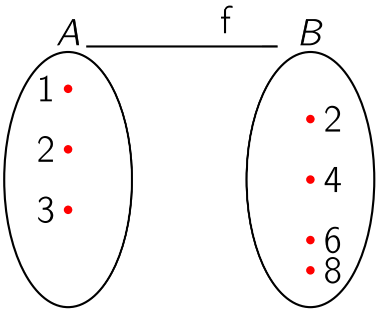
Graphical



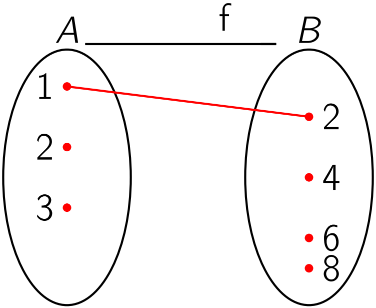
Graphical



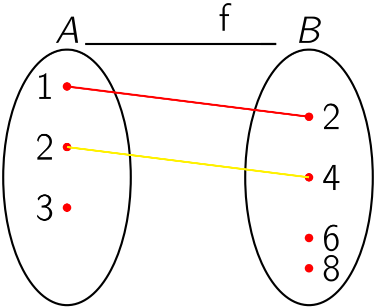
Graphical



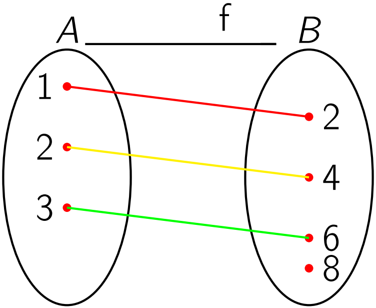
Graphical



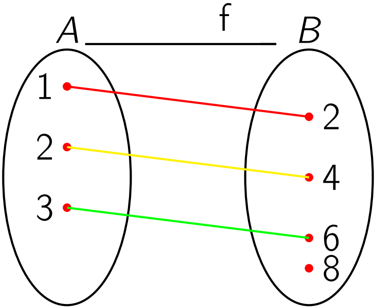
Graphical



Graphical

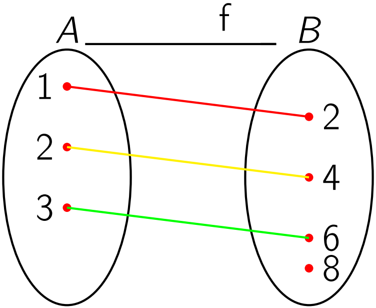


Graphical



Is it function ? If it is, what type is it?

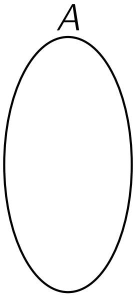
Graphical



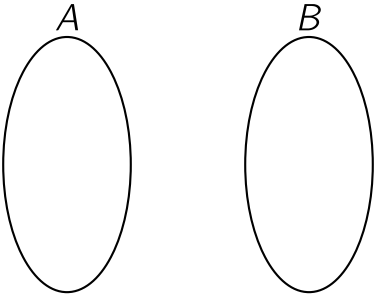
Is it function ? If it is, what type is it?

One - to - one (or) Injective

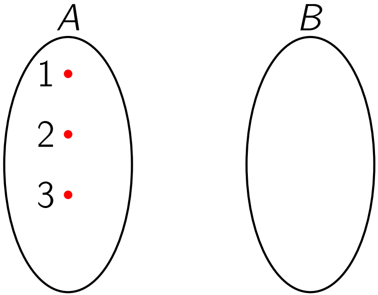
Graphical



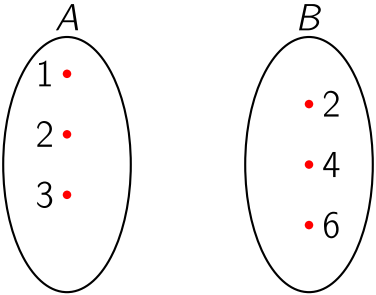
Graphical



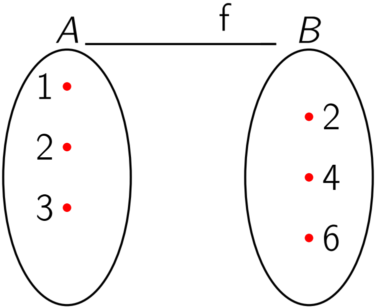
Graphical



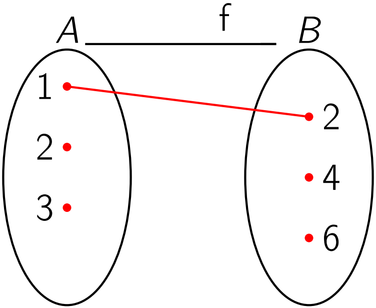
Graphical



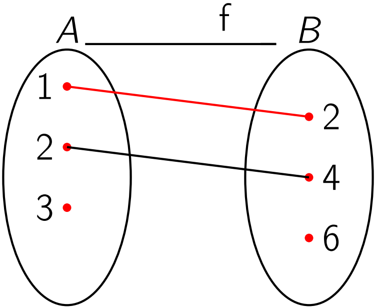
Graphical



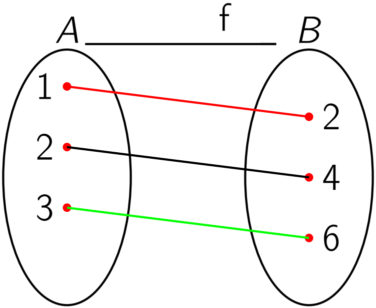
Graphical



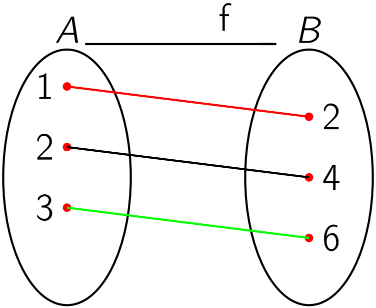
Graphical



Graphical

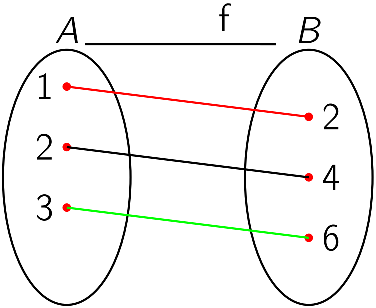


Graphical



Is it function ? If it is, what type is it?

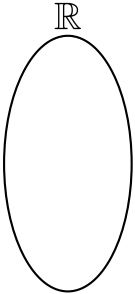
Graphical



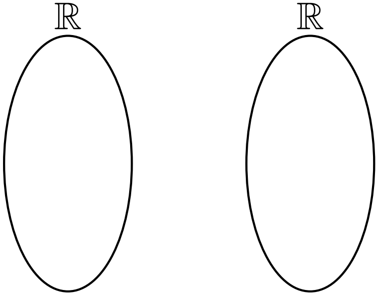
Is it function ? If it is, what type is it?

Onto (or) Surjective

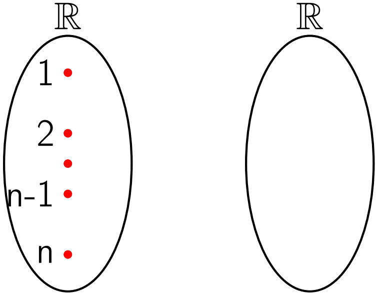
Graphical



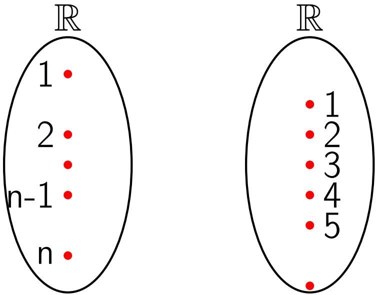
Graphical



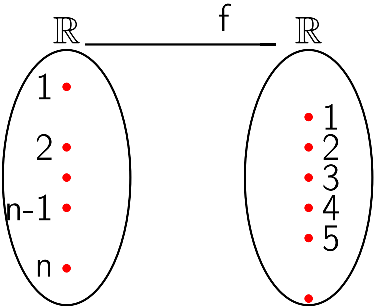
Graphical



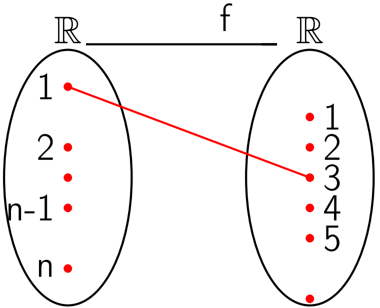
Graphical



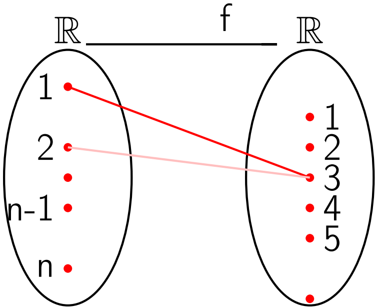
Graphical



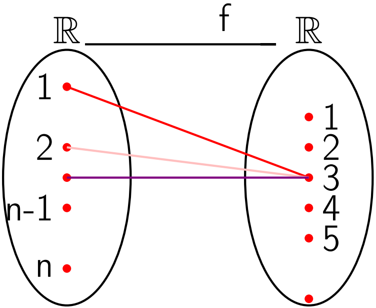
Graphical



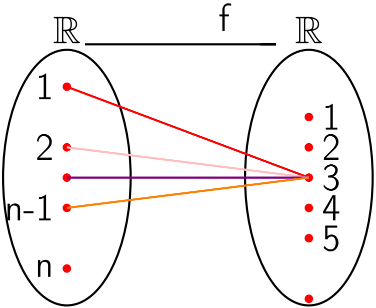
Graphical



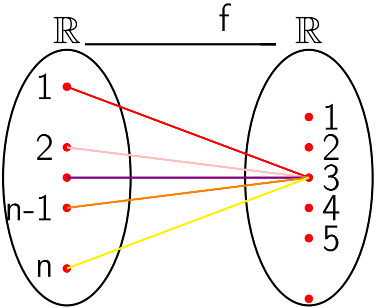
Graphical



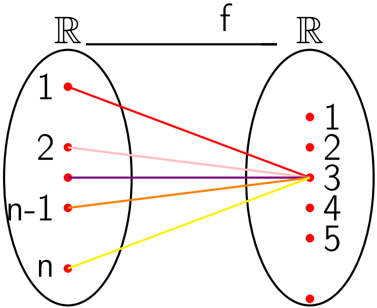
Graphical



Graphical



Graphical



Constant Function

$f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 3$ is called a constant function. The range of f is 3.

Introducing Sequence

☞ In maths, we call a list of numbers in order a sequence.

Introducing Sequence

- In maths, we call a list of numbers in order a sequence.
- Each number in a sequence is called a term.

Introducing Sequence

- In maths, we call a list of numbers in order a sequence.
- Each number in a sequence is called a term.
- If terms are next to each other they are referred to as consecutive terms.

Introducing Sequence

- In maths, we call a list of numbers in order a sequence.
- Each number in a sequence is called a term.
- If terms are next to each other they are referred to as consecutive terms.
- When we write out sequences, consecutive terms are usually separated by commas.

Example

Consider the following collection of real numbers given by

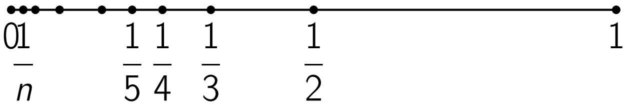
$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$$

Example

Consider the following collection of real numbers given by

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$$

Graphical

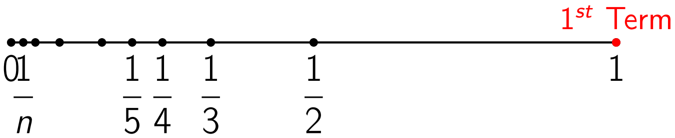


Example

Consider the following collection of real numbers given by

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$$

Graphical

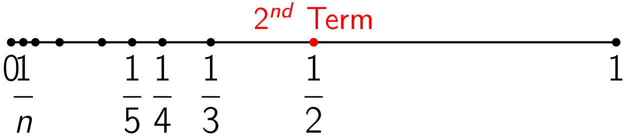


Example

Consider the following collection of real numbers given by

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$$

Graphical

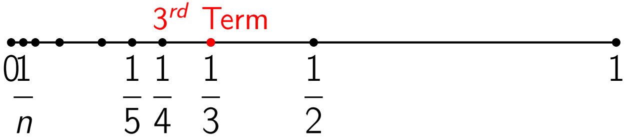


Example

Consider the following collection of real numbers given by

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$$

Graphical

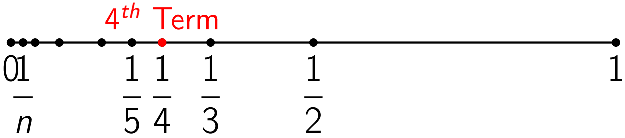


Example

Consider the following collection of real numbers given by

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$$

Graphical

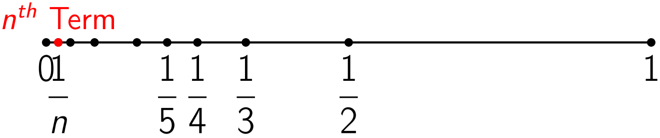


Example

Consider the following collection of real numbers given by

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$$

Graphical

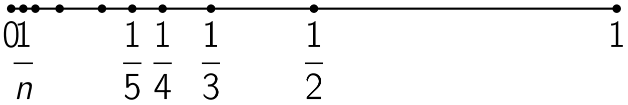


Example

Consider the following collection of real numbers given by

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$$

Graphical



This is an example of sequence of real numbers.

Sequence is a function whose domain is the set of natural numbers.

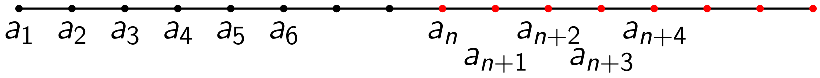
Sequence is a function whose domain is the set of natural numbers.

Definition

Let $f : \mathbb{N} \to \mathbb{R}$ be a function and $f(n) = a_n$. Then $a_1, a_2, a_3, \cdots, a_n, \cdots$, is called the sequence in $\mathbb{R}$ determined by the function f and is denoted by $\{a_n\}$, a_n is called the n^{th} term of the sequence.

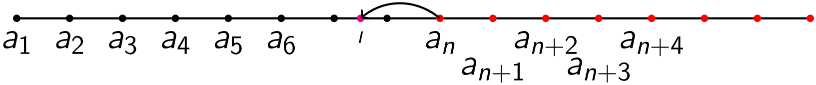
Convergence of a Sequence

We say that a sequence (x_n) converges if there exists $x_0 \in \mathbb{R}$ such that for every $\epsilon > 0$, there exists a positive integer N (depending on ϵ) such that $x_n \in (x_0 - \epsilon, x_0 + \epsilon)$ for all $n \geq N$.



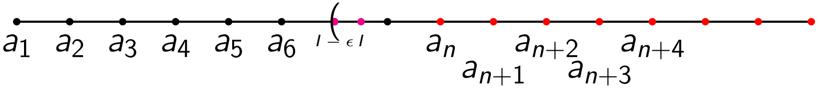
Definition

Let $\{a_n\}$ be a sequence of real numbers.



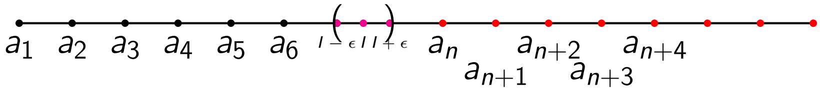
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$



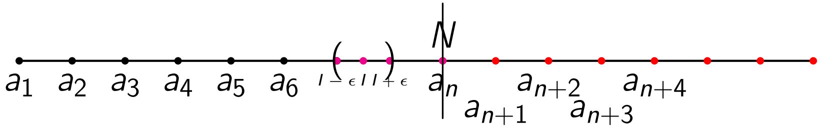
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$



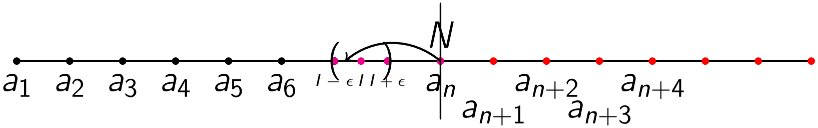
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$



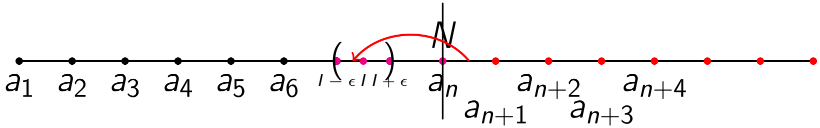
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$ there exists a natural number N



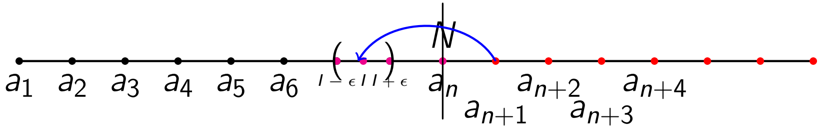
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$ there exists a natural number N such that $a_n \in (l-\epsilon, l+\epsilon)$



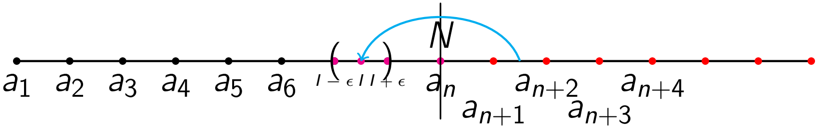
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$ there exists a natural number N such that $a_n \in (l-\epsilon, l+\epsilon)$ for all $n \geq N$.



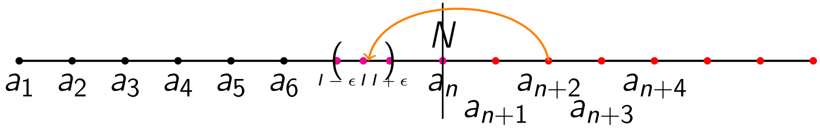
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$ there exists a natural number N such that $a_n \in (l - \epsilon, l + \epsilon)$ for all $n \geq N$.



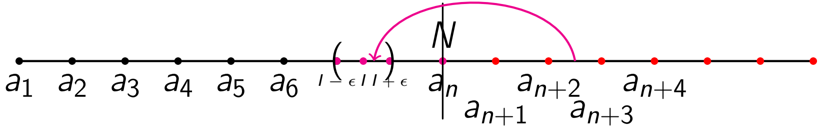
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$ there exists a natural number N such that $a_n \in (l-\epsilon, l+\epsilon)$ for all $n \geq N$.



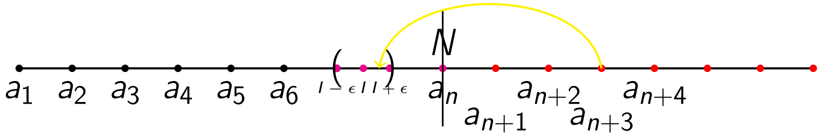
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$ there exists a natural number N such that $a_n \in (l-\epsilon, l+\epsilon)$ for all $n \geq N$.



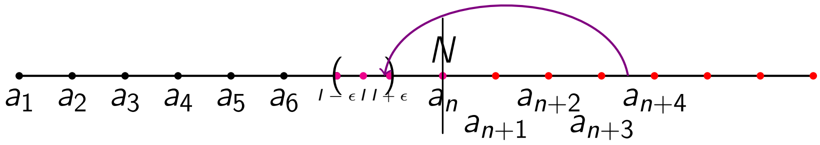
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$ there exists a natural number N such that $a_n \in (l-\epsilon, l+\epsilon)$ for all $n \geq N$.



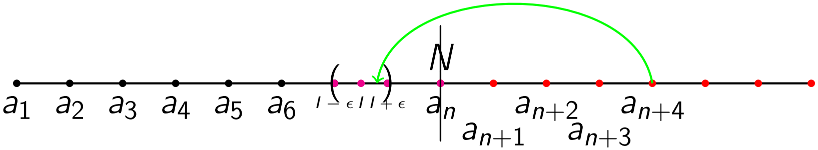
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$ there exists a natural number N such that $a_n \in (l-\epsilon, l+\epsilon)$ for all $n \geq N$.



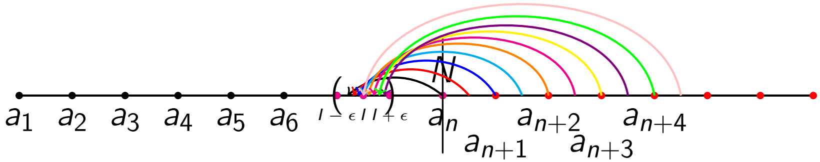
Definition

Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$ there exists a natural number N such that $a_n \in (l-\epsilon, l+\epsilon)$ for all $n \geq N$.



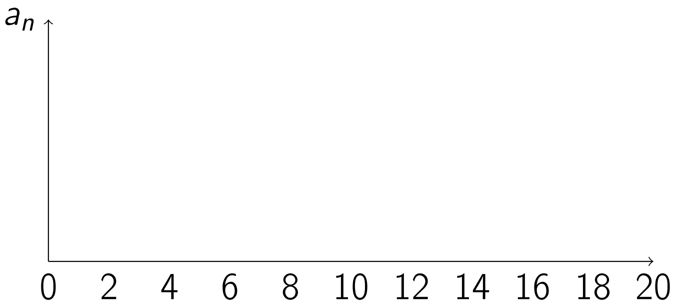
Definition

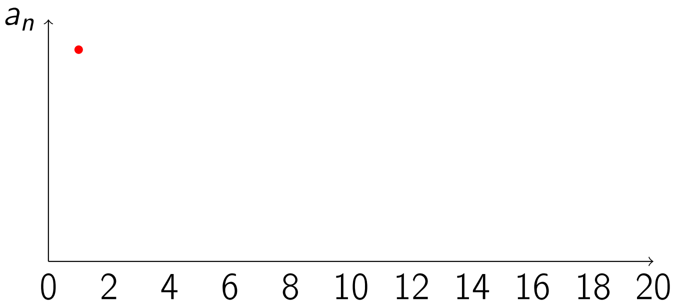
Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$ there exists a natural number N such that $a_n \in (l-\epsilon, l+\epsilon)$ for all $n \geq N$.

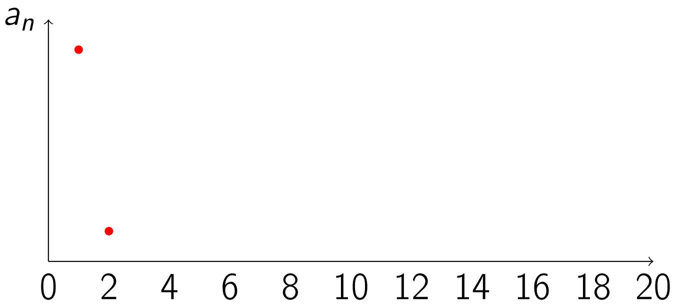


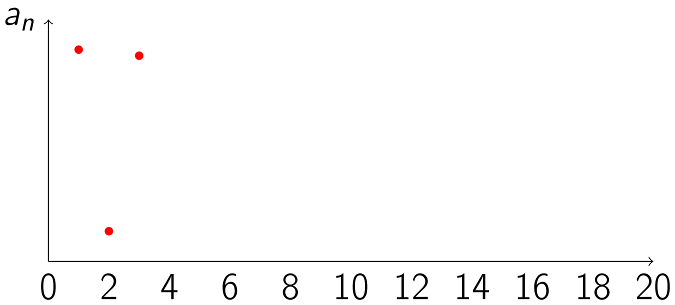
Definition

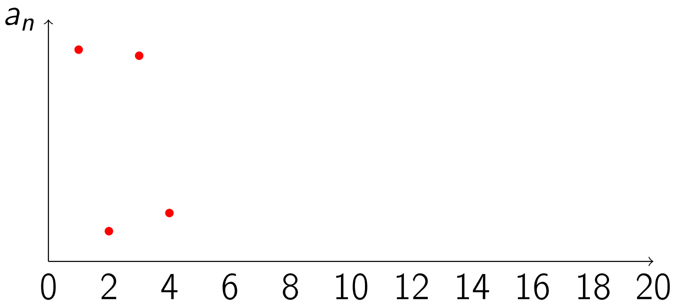
Let $\{a_n\}$ be a sequence of real numbers. $\{a_n\} \to l$ iff given $\epsilon > 0$ there exists a natural number N such that $a_n \in (l-\epsilon, l+\epsilon)$ for all $n \geq N$.

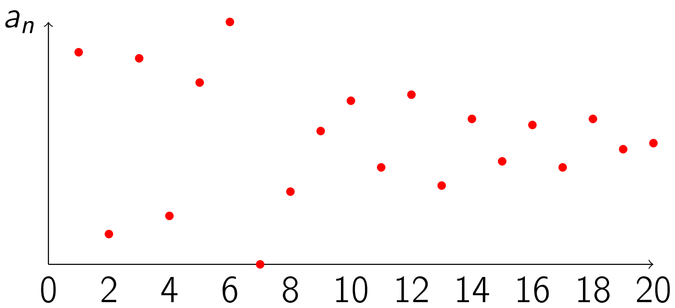

a_n
0 2 4 6 8 10 12 14 16 18 20

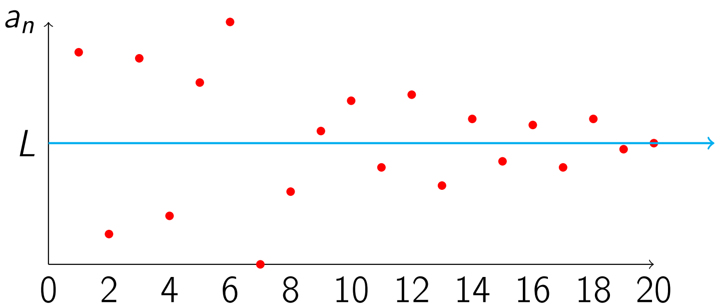

a_n
0 2 4 6 8 10 12 14 16 18 20

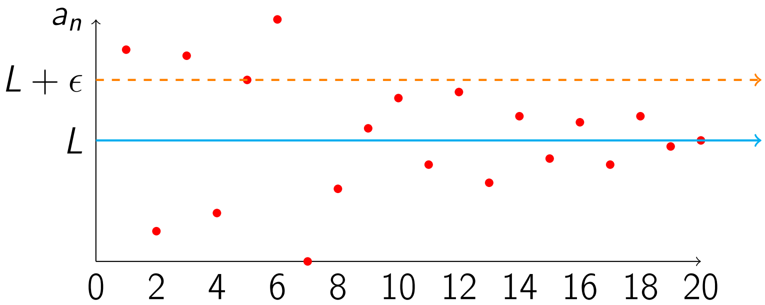

a_n
0
2
4
6
8
10
12
14
16
18
20

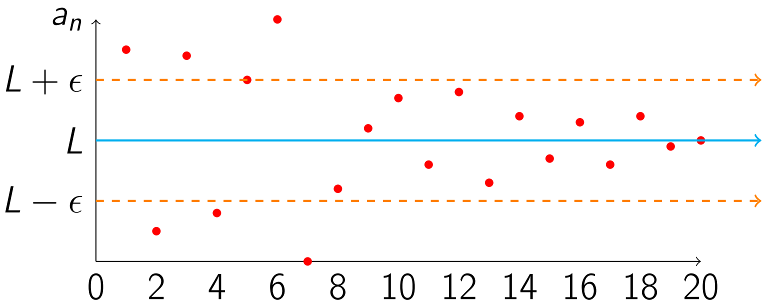

a_n
0
2
4
6
8
10
12
14
16
18
20

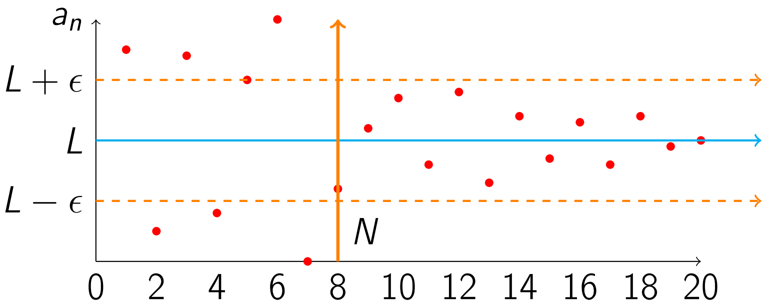

a_n
0 2 4 6 8 10 12 14 16 18 20

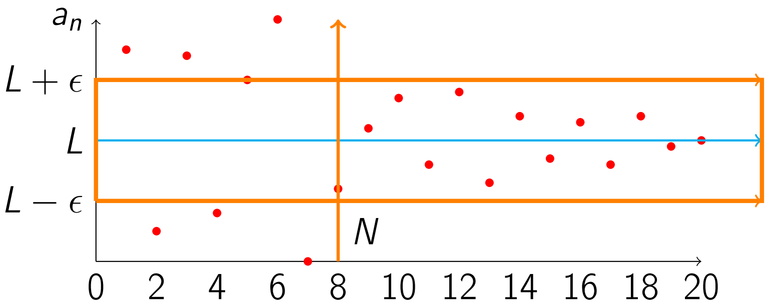

a_n
0
2
4
6
8
10
12
14
16
18
20

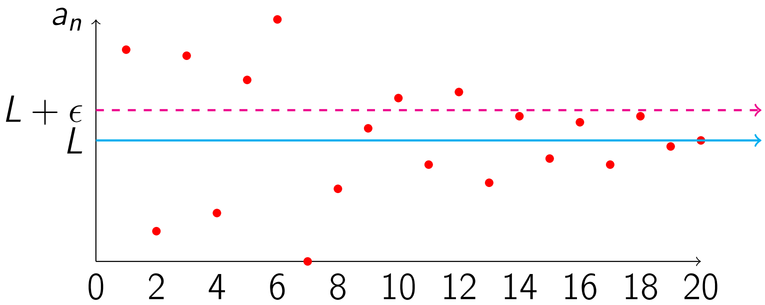

a_n
L
0 2 4 6 8 10 12 14 16 18 20

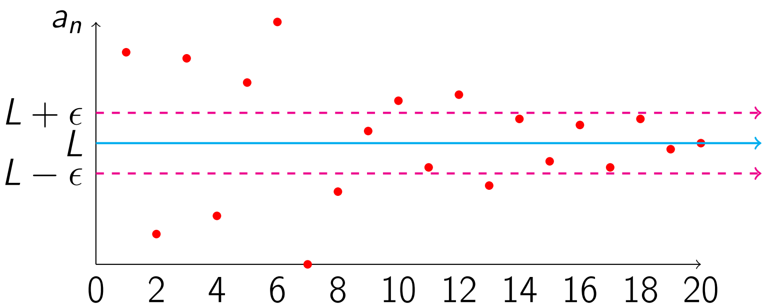

a_n
$L + \epsilon$
L
0 2 4 6 8 10 12 14 16 18 20

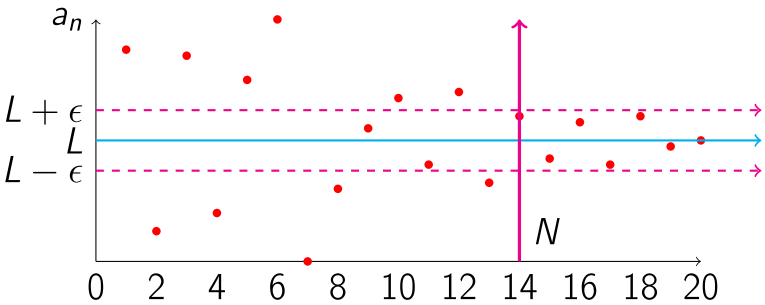

a_n
$L+\epsilon$
L
$L-\epsilon$
0 2 4 6 8 10 12 14 16 18 20

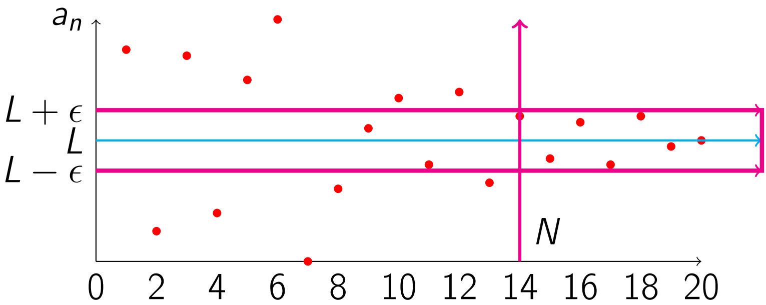

a_n
$L+\epsilon$
L
$L-\epsilon$
N
0 2 4 6 8 10 12 14 16 18 20


a_n
$L + \epsilon$
L
$L - \epsilon$
N
0 2 4 6 8 10 12 14 16 18 20

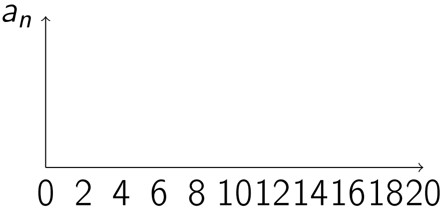

a_n
$L+\epsilon$
L
0
2
4
6
8
10
12
14
16
18
20


a_n
$L+\epsilon$
L
$L-\epsilon$
0 2 4 6 8 10 12 14 16 18 20

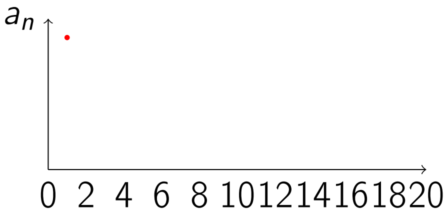

a_n
$L+\epsilon$
L
$L-\epsilon$
N
0 2 4 6 8 10 12 14 16 18 20


a_n
$L + \epsilon$
L
$L - \epsilon$
N
0 2 4 6 8 10 12 14 16 18 20

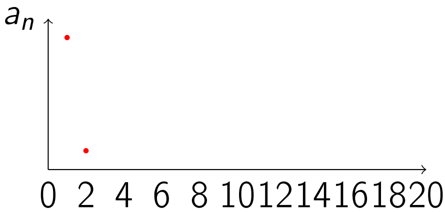
Graphical View



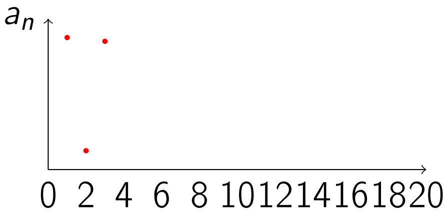
Graphical View



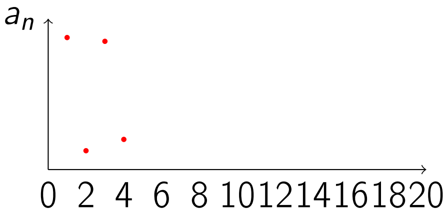
Graphical View



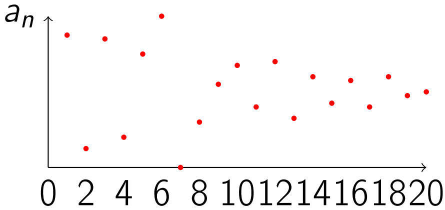
Graphical View



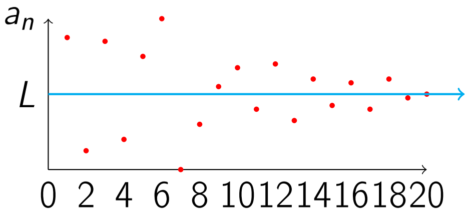
Graphical View



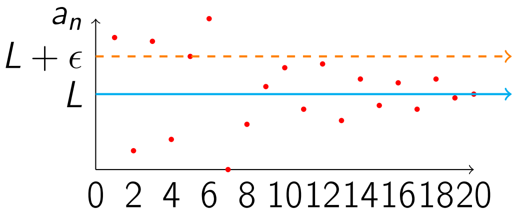
Graphical View



Graphical View

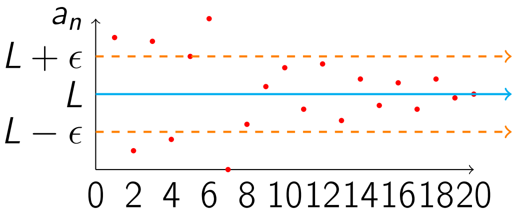


Graphical View



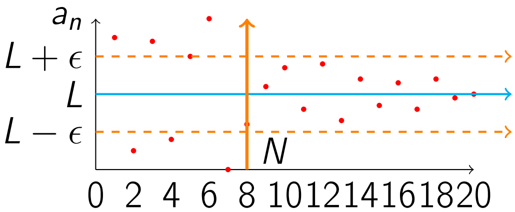
For any $\epsilon > 0$,

Graphical View



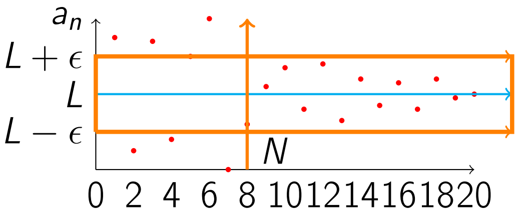
For any $\epsilon > 0$,

Graphical View



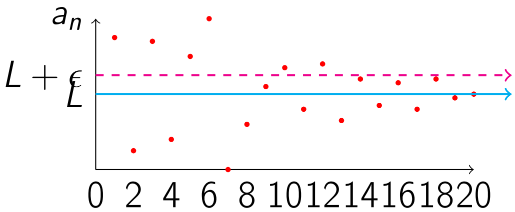
For any $\epsilon > 0$, $\exists$ a positive integer N

Graphical View



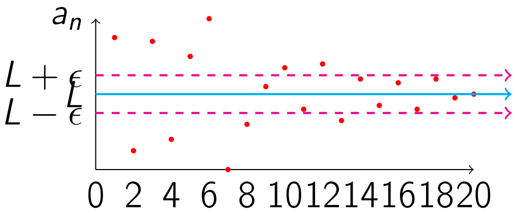
For any $\epsilon > 0$, $\exists$ a positive integer N such that $|a_n - L| \leq \epsilon$ for all $n > m$.

Graphical View



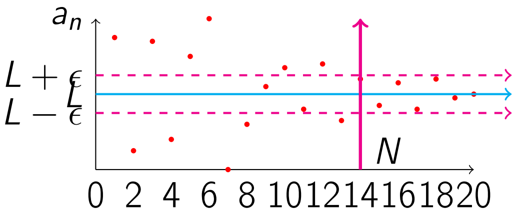
For any $\epsilon > 0$, $\exists$ a positive integer N such that $|a_n - L| \leq \epsilon$ for all $n > m$.

Graphical View



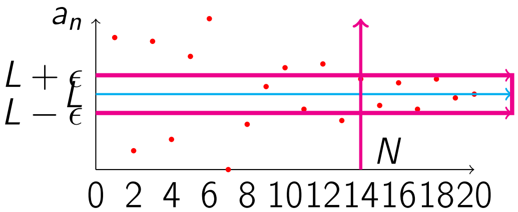
For any $\epsilon > 0$, $\exists$ a positive integer N such that $|a_n - L| \leq \epsilon$ for all $n > m$.

Graphical View



For any $\epsilon > 0$, $\exists$ a positive integer N such that $|a_n - L| \leq \epsilon$ for all $n > m$.

Graphical View



For any $\epsilon > 0$, $\exists$ a positive integer N such that $|a_n - L| \leq \epsilon$ for all $n > m$.

Properties of sequence

1. A sequence cannot converge to two different limits.

Properties of sequence

1. A sequence cannot converge to two different limits.
2. A sequence converges to real number A and B then $A = B$.

Properties of sequence

1. A sequence cannot converge to two different limits.
2. A sequence converges to real number A and B then $A = B$.
3. Any convergent sequence is a bounded sequence.

Properties of sequence

1. A sequence cannot converge to two different limits.
2. A sequence converges to real number A and B then $A = B$.
3. Any convergent sequence is a bounded sequence. Converse is not true.

Properties of sequence

1. A sequence cannot converge to two different limits.
2. A sequence converges to real number A and B then $A = B$.
3. Any convergent sequence is a bounded sequence. Converse is not true. Example : $\{(-1)^n\}$ is a bounded sequence but not a convergent sequence.

Properties of sequence

1. A sequence cannot converge to two different limits.
2. A sequence converges to real number A and B then $A = B$.
3. Any convergent sequence is a bounded sequence. Converse is not true. Example : $\{(-1)^n\}$ is a bounded sequence but not a convergent sequence.
4. Any convergent sequence is bounded.

Concept

Continuous functions are functions that take nearby values at nearby points.

Origin

- The term continuous has been used since the time of Newton to refer to the motion of bodies or to describe an unbroken curve

Origin

- The term continuous has been used since the time of Newton to refer to the motion of bodies or to describe an unbroken curve
- It was made precise until the Nineteenth century.

Origin

- The term continuous has been used since the time of Newton to refer to the motion of bodies or to describe an unbroken curve
- It was made precise until the Nineteenth century.
- Work of Bernard Bolzano in 1817 and Cauchy 1821 identified continuity as a very significant property of function

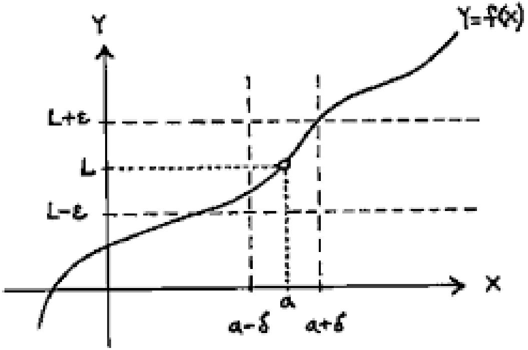
Origin

- The term continuous has been used since the time of Newton to refer to the motion of bodies or to describe an unbroken curve
- It was made precise until the Nineteenth century.
- Work of Bernard Bolzano in 1817 and Cauchy 1821 identified continuity as a very significant property of function
- The concept is tied to that of limit, it was the careful work of Weierstrassin the 1870s that brought proper understanding to the idea of continuity.

Continuous function

Let $f : A \longrightarrow R$, where $A \subset R$, and suppose that $c \in A$. Then f is continuous at c if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $|x - c| < \delta$ and $x \in A$ implies that $|f(x) - f(c)| < \varepsilon$.

Graph



Note

A function $f : A \longrightarrow R$ is continuous on a set $B \subset A$ if it is continuous at every point in B, and continuous if it is continuous at every point of its domain.

Steps

1. Take $|f(x) - f(c)| < \varepsilon$ and rewrite it to match $|x - c| < \delta$ to create a direct relationship

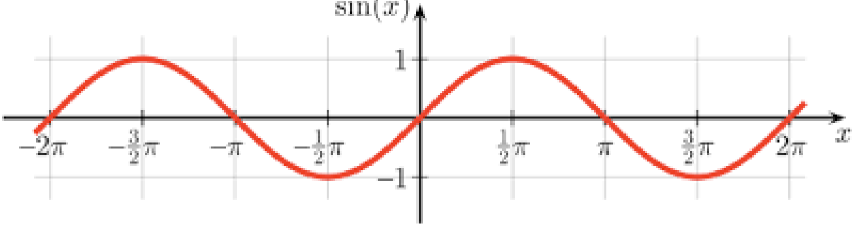
Steps

1. Take $|f(x) - f(c)| < \varepsilon$ and rewrite it to match $|x - c| < \delta$ to create a direct relationship
2. Let $|x - c| < \delta$ and prove $|f(x) - f(c)| < \varepsilon$

Continuous function

The function $sinx : R \longrightarrow R$is continuous on R.

Sinx curve



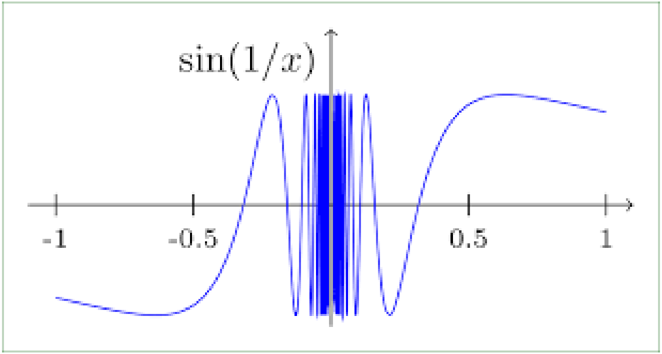
Continuous function

Choose $\delta = \varepsilon$ in the definition of continuity for every $c \in R$

Continuous function

The function $f : R \longrightarrow R$ defined by $f(x) = sin(1/x)$, if $x \neq 0$, $f(x) = 0$,if $x = 0$ is continuous on $R - 0$, since it is the composition of $x \mapsto 1/x$, which is continuous on $R - 0$ and $y \mapsto siny$, which is continuous on R.

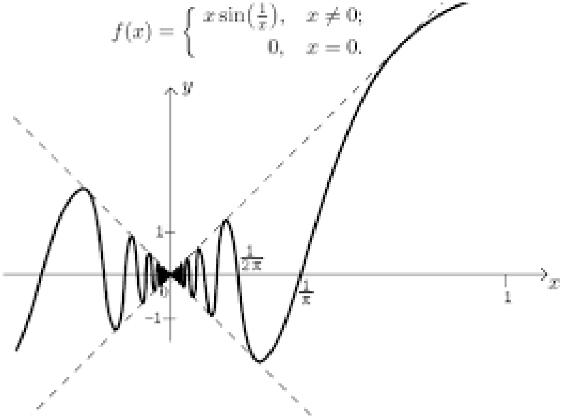
$Sin(\frac{1}{x})$ curve



Continuous function

The function $f : R \longrightarrow R$ defined by $f(x) = xsin(1/x), ifx \neq 0, f(x) = 0, ifx = 0$. Then f is continuous at 0.

$xSin(\frac{1}{x})$ curve



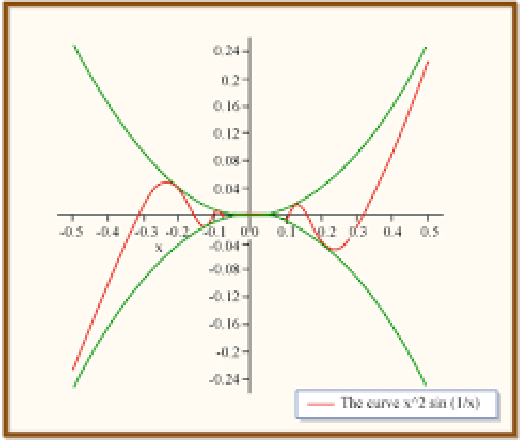
Continuous function

The function $f : R \longrightarrow R$ defined by $f(x) = xsin(1/x), ifx \neq 0, f(x) = 0, ifx = 0.$ Then f is continuous on R 0

Continuous function

The function $f : R \longrightarrow R$ defined by $f(x) = x^2 sin(1/x), if x \neq 0, f(x) = 0, if x = 0.$ Then f is continuous at 0.

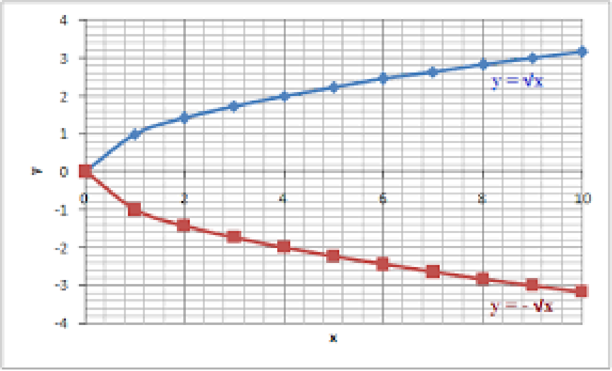
$x^2 Sin(\frac{1}{x})$ curve



Continuous function

The function $f : [0, \infty) \longrightarrow R$ defined by $f(x) = \sqrt{x}$ is continuous on $[0, \infty)$. (i) Prove that f is continuous at c > 0, we can choose $\delta = \sqrt{c}\varepsilon > 0$
(ii) Prove that f is continuous at 0, we note that if $0 \leq x < \delta$ where $\delta = \varepsilon^2 > 0$,

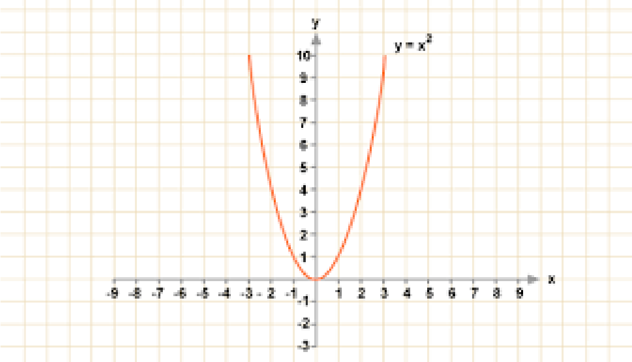
$$f(x) = \sqrt{x} \text{ curve}$$



Continuous function

The function $f(x) = x^2 + 1$ is continuous at $x = 2$

x^2 curve



Uniform Continuous function

Let $f : A \longrightarrow R$, where $A \subset R$. Then f is uniformly continuous on A if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $|x - y| <$ and $x, y \in A$ implies that $|f(x) - f(y)| < \varepsilon$.

Remarks

- The key point of this definition is that δ depends only on ε, not on x, y.

Remarks

- The key point of this definition is that δ depends only on ε, not on x, y.
- A uniformly continuous function on A is continuous at every point of A, but the converse is not true.

Remarks

- The key point of this definition is that δ depends only on ε, not on x, y.
- A uniformly continuous function on A is continuous at every point of A, but the converse is not true.

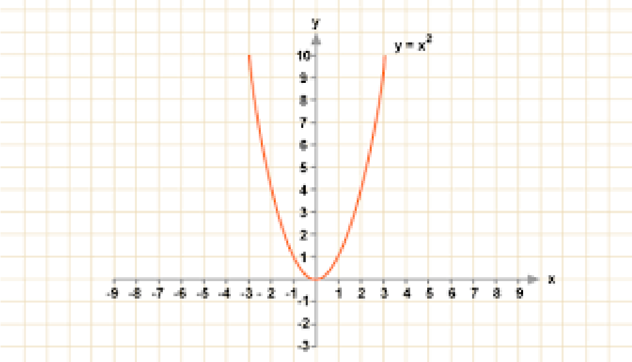
Continuous function

The sine function is uniformly continuous on R, since we can take $\delta = \varepsilon$ for every $x, y \in R$.

Continuous function

Define $f : [0, 1] \longrightarrow R$ by $f(x) = x^2$. Then f is uniformly continuous on $[0, 1]$.

x^2 curve



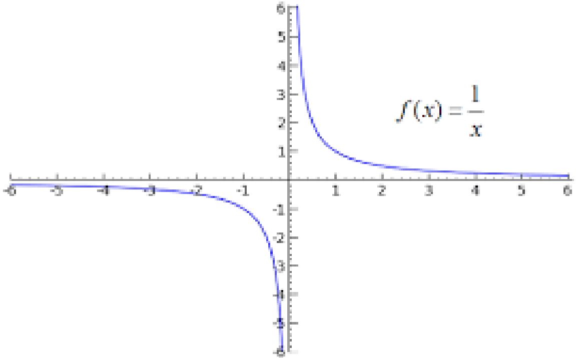
Continuous function but not uniform

The function $f(x) = x^2$ is continuous but not uniformly continuous on R.

Continuous function

The function $f : (0, 1] \longrightarrow R$ defined by $f(x) = \frac{1}{x}$ is continuous but not uniformly continuous on $(0, 1]$.

$\left(\frac{1}{x}\right)$ curve



Continuous function but not uniform

Define $f : (0,1] \longrightarrow R$ by $f(x) = sin(\frac{1}{x})$
Then f is continuous on $(0,1]$ but it is not uniformly continuous on $(0,1]$.

❦ ❦ Time to Interact ❦ ❦

Thank You