TRANSPORTATION PROBLEM

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Finding Initial Basic Feasible Solution(IBFS)

Introduction

- The transportation problem is a special type of linear programming problem.
- The objective is to minimize the cost of distributing a product from a number of sources to a number of destinations.

Basic Definition:

Feasible Solution:

- A solution that satisfies the row and column sum restrictions and also the non-negativity restrictions is a feasible solution.

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Basic Feasible Solution:

A feasible solution of (m X n) transportation problem is said to be basic feasible solution, when the total number of allocations is equal to (m + n - 1).

Optimal Solution:

A feasible solution is said to be optimal solution when the total transportation cost will be the minimum cost.

Balanced Transportation Problem:

If total supply = total demand then it is a balanced transportation problem.

There will be (m + n - 1) basic independent variables out of (m x n) variables.

Methods for finding an initial basic feasible solution:

- North West Corner Rule(NWCR)
- Least Cost Method (or)Matrix Minimum Method(LCM)
- Vogel's Approximation Method(VAM)

North West Corner Rule(NWCR)

(i) Formulate the given problem as LPP and set up the problem in the tabular form known as transportation table.

(ii) Select the North-west (i.e., upper left) corner cell of the table and allocate the maximum possible units between the supply and demand requirements. During allocation, the transportation cost is completely discarded (not taken into consideration).

(iii) Delete that row or column which has no values (fully exhausted) for supply or demand.

(iv) (iv) Now, with the new reduced table, again select the North-west corner cell and allocate the available values.

(v) Repeat steps (ii) and (iii) until all the supply and demand values are zero.

(vi) Obtain the initial basic feasible solution.

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- **Example:** Obtain an Initial Basic Feasible Solution to the following transportation problem using the North-West corner method.

Source	Destination				Supply
	1	2	3	4	
1	11	13	17	14	250
2	16	18	14	10	300
3	21	24	13	10	400
Demand	200	225	275	250	950/950

Cont……

- Solution:
- Since $\sum a_i = \sum b_j = 950$, the given problem is a balanced one. There exists a feasible solution to the transportation problem which can be solved by North-West corner method.
- The transportation table of the given problem contains 12 cells. Select the North- West corner cell (1, 1) to make the first allocation. The corresponding supply and demand values are 250 and 200 respectively.
- Allocate the maximum possible value to satisfy the demand from the supply, so allocate 200 to the cell (1, 1) as shown below,

	1	2	3	4	Supply
1	200 / 11	13	17	14	250
2	16	18	14	10	300
3	21	24	13	10	400
Demand	200	225	275	250	

Cont……

- Now delete the column one which is exhausted and gives a new reduced table as shown below

	2	3	4	Supply
1	[50] 13	17	14	50
2	18	14	10	300
3	24	13	10	400
Demand	225	275	250	

- From the above table after deleting row 1 it is given that,

	2	3	4	Supply
2	[175] 18	14	10	300
3	24	13	10	400
Demand	175	275	250	

Cont……

- From the above table after deleting column 2 it is given that,

	3	4	Supply
2	125 14	10	300
3	13	10	400
Demand	275	250	

- Finally, row 2 sources 3 is left allocate to destination 3 and 4 satisfies the supply of 400.

	3	4	Supply
3	150 13	250 10	400
Demand	150	250	

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- The Initial Basic Feasible Solution using the North-West corner method is shown below,

Source	1	2	3	4	Supply
1	200 / 11	50 / 13	17	14	250
2	16	175 / 18	125 / 14	10	300
3	21	24	150 / 13	250 / 10	400
Demand	200	225	275	250	

- The transportation cost =(200*11)+(50*13)+(175*18)+(125*14)+(125*13)+(250*10)

= Rs.12,200 /-

LEAST COST METHOD/ MATRIX MINIMA METHOD(LCM)

Steps

(i) Select the smallest transportation cost cell available in the entire table and allocate the supply and demand.

(ii) Delete that row/column which has exhausted. The deleted row/column must not be considered for further allocation.

(iii) Again select the smallest cost cell in the existing table and allocate. (Note: In case, if there are more than one smallest cost, select the cells where maximum allocation can be made)

(iv) Obtain the initial basic feasible solution.

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- **Example:** Obtain an Initial Basic Feasible Solution to the following transportation problem using the Least-Cost method.

Source	Destination				Supply
	1	2	3	4	
1	11	13	17	14	250
2	16	18	14	10	300
3	21	24	13	10	400
Demand	200	225	275	250	950/950

Cont……

- Solution: Since $\sum a_i = \sum b_j = 950$ the given problem is a balanced one. There exists a feasible solution to the transportation problem which can be solved by Least-Cost method.
- The transportation table of the given problem contains 12 cells. Select the minimum cost cell from the table (2, 4) and (3, 4) cell which is a tie. If there is a tie, it is preferable to select a cell to which maximum allocation can be made. In this case the maximum allocation is 400 which is made in the cell (3, 4). The corresponding supply and demand values are 250 and 400 respectively.
- Allocate the maximum possible value to satisfy the demand from the supply, so allocate 250 to the cell (3, 4) as shown below,

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	1	2	3	4	Supply
1	11	13	17	14	250
2	16	18	14	10	300
3	21	24	13	250 10	400
Demand	200	225	275	250	

- Now delete the column 4 which is exhausted and give a new reduced table. Take again the next minimum cost value available in the table (1, 2) cell and allocate the value of 200 as shown below,

	1	2	3	Supply
1	200 11	13	17	250
2	16	18	14	300
3	21	24	13	150
Demand	200	225	275	

- In the reduced table the minimum cost is 13 which occurs in two cells namely (1, 2) and (3, 3) the maximum allocation may be done in (1, 2).

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	2	3	Supply
1	50 13	17	50
2	18	14	300
3	24	13	150
Demand	225	275	

- after deleting row 1 from the above table, the reduced matrix is given by,

	2	3	Supply
2	18	14	300
3	24	150 13	150
Demand	175	275	

- Finally, column 2, source 3 is left. Allocate to destination 2 and 3 satisfies the supply of 300.

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	2	3	Supply
3	175 (24)	125 (13)	300
Demand	175	125	

- The Initial Basic Feasible Solution using the Least-Cost method is thus shown below,

Source	1	2	3	4	Supply
1	200 (11)	50 (13)	17	14	250
2	16	175 (18)	125 (14)	10	300
3	21	24	150 (13)	250 (10)	400
Demand	200	225	275	250	

- The transportation cost = (200*11)+(50*13)+(175*18)+(125*14)+(125*13)+(250*10)

= Rs.12,200 /-

Vogel's Approximation Method(Steps)

(i) Calculate penalties for the each row and column by taking the difference between the smallest cost and next highest cost available in that row/column. If there are two smallest costs, then the penalty is zero.

(ii) Select the row/column, which has the largest penalty and make allocation in the cell having the least cost in the selected row/column. If two or more equal penalties exist, select one where a row/column contains minimum unit cost. If there is again a tie, select one where maximum allocation can be made.

(iii) Delete the row/column, which has satisfied the supply and demand.

(iv) Repeat steps (i) and (ii) until the entire supply and demands are satisfied.

(v) Obtain the initial basic feasible solution.

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- **Example:** Obtain an Initial Basic Feasible Solution to the following transportation problem using the Vogel's Approximation method.

Source	Destination				Supply
	1	2	3	4	
1	11	13	17	14	250
2	16	18	14	10	300
3	21	24	13	10	400
Demand	200	225	275	250	950/950

Cont……

Solution:

- Since $\sum a_i = \sum b_j = 950$ the given problem is a balanced one. There exists a feasible solution to the transportation problem which can be solved by Vogel's Approximation method.
- The transportation table of the given problem contains 12 cells.
- Find the penalties for each row and column. Choose the row/column, which has the maximum value for allocation.

	1	2	3	4	Supply	Penalty
1	[200] 11	13	17	14	250	(2)
2	16	18	14	10	300	(4)
3	21	24	13	10	400	(3)
Demand	200	225	275	250		
Penalty	(5) ↑	(5)	(1)	(0)		

Cont……

- In the above case we have two penalties, select the least cost which is in row 1 and hence select the (1, 1) for allocation. The supply and demand are 200 and 250 respectively and hence allocate 200 in the cell as shown above.
- Now delete the column one which is exhausted and again calculate the penalties for the remaining row and column.
- The new reduced table is given below:

	2	3	4	Supply	Penalty
1	50 13	17	14	50	(1)
2	18	14	10	300	(4)
3	24	13	10	400	(3)
Demand	225	275	250		
Penalty	(5)	(1)	(0)		

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Cont……

- Since the supply is only 50 then delete row 1 and the new reduced matrix is

	2	3	4	Supply	Penalty
2	[175] 18	14	10	300	(4)
3	24	13	10	400	(3)
Demand	175	275	250		
Penalty	(6) ↑	(1)	(0)		

- In this table, the maximum penalty is 6 and demand is 175 allocate in the (2, 2) the reduced matrix is given below,

	3	4	Supply	Penalty	
2	14	[125] 10	125	(4)	←
3	13	10	400	(3)	
Demand	275	250			
Penalty	(1)	(0)			

Cont……

- Finally, after deleting row 2 source 3 is left allocate to destination 3 and 4 it satisfies the supply of 400.

	3	4	Supply
3	275 (13)	125 (10)	400
Demand	275	125	

- The Initial Basic Feasible Solution using the Vogel's Approximation method is shown below,

Source	1	2	3	4	Supply
1	200 (11)	50 (13)	17	14	250
2	16	175 (18)	14	125 (10)	300
3	21	24	275 (13)	125 (10)	400
Demand	200	225	275	250	

- The transportation cost = (200*11)+(50*13)+(175*18)+(125*10)+(275*13)+ (125*10) = Rs.12, 075/-

Thank You