INTRODUCTION TO SET THEORY

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TOPICS

- Basic concepts of Set theory
- Types of Set
- Operations on Sets
- Laws of Set theory (Set Identities)

Basic Definition

- A collection of well defined objects is called a set and described with in braces({}).
- The uppercase English alphabets, with or without subscripts, are used to denote sets and lowercase English alphabets are used for denote objects of the set.
  - E.g. A set of all Alphabets A={ a, b, c,…..,z}
- Any object in the set is called element or member of the set.
- If x is an element of the set A, then we can read as “”x belongs to A” or “x is in A” , and if x is not an element of X, then we can read as “” x does not belongs to A”

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- Typically sets are described by two methods

- Roster or list method:

-In this method, all the elements are listed in braces.

E.g. A = {2, 3, 5, 7, 11, 13 }

- Set-Builder method:

-In this method, elements are described by the property they satisfy.

E.g. A = { x : x is a prime number less than 15}

Cardinality

- The number of elements in the set A is called cardinality of the set A, denoted by |A| or n(A) .
- We note that in any set the elements are distinct. The collection of sets is also a set.

  E.g. S = {2, {3, 5}, 7, 11, 13 }
- Here {3, 5} itself one set and it is one element of S and |S|=4.

Types of Set

Universal set

- A set which contains all objects under consideration is called as Universal set and is denoted by E or U.
- E.g. For example, U = {0,1,2,3,4,5,6,7,8,9} may be considered as a universal set when we consider sets A = {0,1,3,5} and B = {1,4,7}

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- Finite or Infinite
  - A set is called finite if it contains finite number of distinct elements; otherwise, a set is infinite.
    - E.g. A={Charles, Kumar, Mohan, Ravi} is a finite set
    - E.g. B={1,2,3,4,5,……..} is an infinite set

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- Null Set
- A set contains no element is called a null set.
- It is also called an empty set or a void set, or a zero set.
- It is usually denoted by the Phi(∅) or two empty braces({ }).
- For example, the set of prime numbers between 8 and 10 is null set.

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Subset

- A set A is said to be a subset of set B, if every element of A is also an element of B.
- It is denoted by '⊆' $A \subseteq B$.
- E.g. $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 7, 8\}$ Then $A \subseteq B$.
- Note that

  1. A set is subset of itself.
  2. Null set is subset of every set.

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- Superset
  - A set A is said to be a superset of set B, if B is a subset of A.
- It is denoted by $A \supseteq B$.
- E.g. $A = \{ 1, 2, 3, 4, 7, 8 \}$ and $B = \{1, 2, 3, 4 \}$ Then $A \supseteq B$.

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Proper subset

- A set is A is said to be a proper subset of B, if A is a subset of B and there is at least one element in B, which is not an element of A.
- E.g. A = {1, 2, 3, 4 } and B = { 1, 2, 3, 4, 7, 8 }
- Here A is a Proper Subset of B

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Singleton Set

- A set contains only one element is called a singleton set.
- For example, the set of prime numbers between 24 and 30 is a singleton set, its Only element being 29.

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- Power set
- A power set of a set A, denoted by P(A), is set of all subsets of A.
  - E.g. If A = { 1, 2, 3 }, then, P(A) = { Ø, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3},{1, 2, 3}}.
  - Note: If number of elements in A is n, then the number of elements in the power set of A is 2^n.
  - We observe that n(A) = 3 and n(P (A)) = $2^3 = 8$

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- Equal Sets
  - Two sets A and B are said to be equal iff $A \subseteq B$ and $B \subseteq A$.

  i.e., $A = B \Leftrightarrow (A \subseteq B) \wedge (B \subseteq A)$
- E.g. $A=\{1,3,5,8\}$ and $B=\{1,8,5,3\}$ Then $A=B$

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- Disjoint Set
- Two set A and B are called disjoint if and only if ,A and B have no element in common.
- Example
  - A={1,2,3} B={5,7,9} C={3,4}
  - $A \cap B = \emptyset$ $A \cap C = \emptyset$ $B \cap C = \emptyset$
  - A and B are disjoint and B and C also, but A and C are not disjoint.

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- Complement of a set
  - Let A be any set, and E be universal. The relative complement of A in E is called absolute complement or complement of A.
  - The complement of A is denoted by A^C (or) $\bar{A}$ (or)A'
- Example
  - Let E={1,2,3,4,5.,…..} be universal set and A={2,4,6,8,……} be any set in E, then $\bar{A}$={1,3,5,7,……..}

Operations on Sets

Union of two sets

- The union of two sets A and B is the set of all elements which belong to either A or B or both.
- It is denoted by A U B.
- Thus A U B = {x / x Є A or x Є B}
- For example
  - if A = {1, 2, 3} and B = {2, 4, 6}, then

    A U B = {1, 2, 3, 4, 6}.

Cont……

- Intersection of two sets
  - The intersection of two sets A and B is the set of all elements which belong to both A and B.
  - it is denoted by $A \cap B$
- Thus $A \cap B = \{x / x \in A \text{ and } x \in B\}$
- For example, if $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$ Then $A \cap B = \{2\}$

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- Set Difference (Relative complement)
  - The relative complement of a set A in a set B is the set of elements of B which are not the elements of A.
  - It is denoted by B – A.
  - Thus B - A= {x / x Є B and x∉ A}.
  - It is also called the difference between B and A.
  - We observe that A – B = {x / x Є A and x ∉ B}
  - Thus A - B = B – A and in fact (A – B) ∩ (B - A) = ∅ .
  - For example if A = {1, 2, 3} and B ={2,4,6} Then B-A = {4, 6}, while A – B = {1,3 }

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- Symmetric difference
- The symmetric difference or Boolean sum of two sets A and B is the difference set between A U B and A ∩ B.
- It is denoted by A + B (or) A⊕B.
- Thus A+ B = (A U B) – (A ∩ B) = {x / x Є(AU B) and x ∉(A ∩ B)} = (A – B) U (B – A)
- Example
  - If A = {1, 2, 3} and B = {2, 4, 6} Then A U B = {1, 2, 3, 4, 6} and A ∩ B = {2} So that, A+ B = {1, 3, 4, 6}

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- Cartesian product
  - The Cartesian product of two sets A and B is the set of all ordered pairs (a, b) where a Є A and b Є B.
  - It is denoted by A X B.
  - Thus A X B = {(a, b) / a Є A and b Є B}
- Let A = {1, 2} and B = {x, y, z} then
- A X B = {(1, x), (1, y), (1, z), (2, x), (2, y), (2, z)}
- B X A = {(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)}
- A X A = {(1, 1), (1, 2), (2, 1), (2, 2)}
- B X B = {(x, x), (x, y), (x, z), (y, x), (y, y), (y, z), (z, x), (z, y), (z, z)}

Set Identities

$A \cup A = A$ Idempotent laws

$A \cap A = A$

$A \cup B = B \cup A$ Commutative laws

$A \cap B = B \cap A$

$(A \cup B) \cup C = A \cup (B \cup C)$ Associative laws

$(A \cap B) \cap C = A \cap (B \cap C)$

Cont……

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ Distributive laws

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (A \cap B) = A$ Absorption laws

$A \cap (A \cup B) = A$

$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$ De Morgan's laws

$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

Cont……

- $A \cup \emptyset = A$
  $A \cap U = A$ Identity Laws

$A \cup \bar{A} = U$
$A \cap \bar{A} = \emptyset$ Complement Laws

$\bar{\bar{A}} = A$ Double Complement Law

Thank You